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Vertical FDI and Exchange Rate in a Two-country Model

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Abstract

It has long been observed that the change in exchange rate of a country is positively correlated with Foreign Direct Investment (FDI) inflow into its border. This paper provides an explanation of this positive correlation in a general equilibrium framework, examines the implied welfare effects of short run FDI flows, and offers policy suggestions. We examine firms' cross-border production location decisions in an open-economy macroeconomic model, in which both the exchange rate and FDI flow are endogenously determined. It is argued that the change in the relative real wage caused by nominal rigidities in wage-setting is the underlying cause of the observed correlation. Furthermore, it is shown that these short run fluctuations in FDI speed up the convergence of the economy back to its long run equilibrium, but further exacerbate the inefficiencies in aggregate employments in both countries. Welfare analysis shows that when monetary volatility is sufficiently high, prohibiting FDI increases world utility. Yet the first best outcome can be achieved if long term FDI is retained, but short run variations in production location is disallowed. The theory predicts that industries characterized by wider practice of unionization and collective wage bargaining, and industries with higher labor intensities of production should exhibit higher level of exchange rate-FDI correlations. Given that short run firm relocation is shown to magnify business cycle fluctuations, we then analyze how monetary policy can be designed to mitigate the above-mentioned inefficiency. Specifically, we examine the effects of output gap response in a Taylor-type interest rate rule. Previous literature has emphasized that, when real shocks cause the natural level of output to deviate from steady state, responding to output gap causes distortion. We show that when multinationals exist, output gap response is an effective mechanism to discourage temporary firm relocations. We compute and compare the expected world utility under different environment, and conclude that in every situation, a mild output response always improve welfare.

JEL code: F41, F44

1 Introduction

To many developing countries, foreign direct investment (FDI) is an important source of economic growth and development. FDI promotes growth in labor income and facilitates capital accumulation. Furthermore, it is regarded as a more favorable type of capital flow compared to portfolio investment. FDI flows are noted for their stability in contrast to financial investment flows during currency crises[1]. Probably more importantly, they are believed to carry positive externalities of technology spillover, a process through which domestic firms can become more productive and efficient by learning from affiliates of foreign companies[2]. The positive correlation between FDI inflows and the exchange rate in the short run has been well-documented. Since the mid-1980s, a large volume of empirical studies has repeatedly confirmed that a country tends to receive more FDI inflows when its currency depreciates[3]. Earlier theories explaining this correlation typically take

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[1]For example, Lipsey (2001) examines the behavior of U.S. FDI outflows during three currency crises, and finds that FDI flows are much more stable than other types of capital flows.

[2]See, for example, Keller and Yeaple (2003) and Borensztein, De Gregorio and Lee (1998).

[3]This empirical fact is robust for developed countries and developing countries. Section 2.1 below provides a brief review of previous empirical works.

a partial equilibrium approach, in which the change in exchange rate is treated as an independent and exogenous event. This paper provides an explanation to this correlation by incorporating multinational enterprises (MNE) into an open economy macroeconomic framework, in which both the level of FDI flow and the exchange rate are endogenously determined. We then examine the effects of FDI flows in this framework, and show that short run fluctuations in FDI flows have ambiguous welfare effect, as they lead to adverse changes in the aggregate employment levels.

We study firms' production location decisions in a two-country model largely based on Devereux and Engel (2001). The focus of the paper is to study the behavior of vertical FDI flows, meaning firms invest in a foreign country mainly to take advantage of lower production cost abroad, rather than to gain access into the local market. FDI is modeled as firms' location decisions to be made one period ahead of actual production, and we assume that a fixed sunk cost must be incurred if a firm decides to relocate abroad. The two countries are asymmetric in terms of the average productivity of firms. This asymmetry determines the long run direction of FDI flows. In the model, households set wage in a Calvo manner, inducing sluggish adjustment of the aggregate wages. The uncertainty in the model comes from money supply shocks, which, combined with the nominal friction, cause the short run co-movements of FDI flows and the exchange rate. We assume that firms in each country are heterogeneous in their productivity levels to replicate the empirical finding that MNEs are the more productive firms.

By first analyzing the steady state of the model, we show that productivities determine the long run direction and volume of cross-border FDI flow. Specifically, the country with higher average productivity becomes the source country of FDI, and the country with lower productivity the recipient country. This is because without cross-border relocation of production, the country with lower productivity has lower real wage, making it profitable for the more productive firms in the source country to relocate. We prove that there is always positive long run FDI flow for any finite foreign investment cost.

In the short run, monetary shocks translate into changes in the relative real wage, altering the marginal firm's profitability of relocation, and thus induce fluctuations in FDI flows. The model features frictionless trade of final products, and thus a firm's unit price is identical in both countries when expressed in a common currency. Then a firm's relative profitability of producing abroad depends on its firm-specific productivity, and on the expected relative real wage. A firm decides its production location by comparing the net gain from switching to a low-production-cost location with the sunk cost the switch imposes. We show that given a lower expected real wage abroad, there exists a cutoff firm who will just break even in expectation by switching production location. Every firm with a higher productivity than the cutoff firm must earn a strictly positive profit abroad, and thus becomes an MNE in the following period. Therefore, the cutoff productivity pins down the mass of MNEs and the volume of FDI. When a given monetary shock hits, the relative aggregate price changes to fully reflect the change in the relative money supply. However, as only a fraction of households can update their wage contracts in any given period, the aggregate wage rate only adjust partially in response to the monetary shocks. As a result, a monetary expansion in the foreign country causes a proportional rise of the relative foreign aggregate price and a proportional depreciation of the foreign currency. Yet under nominal wage rigidity, the rise in the foreign aggregate wage rate only captures part of the increase in the aggregate price, causing an actual depreciation of the foreign real wage. The depreciation of the foreign real wage then attracts more home firms to relocate abroad, generating increased FDI flow from the home country to foreign. As such, we are able to provide an explanation for the positive correlation between FDI inflows and the exchange rate in the short run.

By contrasting the dynamics of the real wages and aggregate employment in the model with the dynamics under the scenario of domestic production, we show that the short run impacts of production relocation on the labor markets of the two countries lead to ambiguous welfare effects. On the one hand, production relocation speeds up the adjustment of the relative real wage and aggregate employment back to their long run equilibrium levels, partially counteracting the sluggish effect of the Calvo wage-setting. On the other hand, this process is achieved by further distorting the aggregate employment levels. Under a foreign monetary expansion, for example, the foreign real wage depreciates against that of the home country. The relocation of home firms abroad leads to an increased labor demand in the foreign country, causing a faster adjustment of the wage rate. However, under domestic production, foreign households are already working too much compared with the flexible-price long run equilibrium level. Yet when cross-border relocation is allowed, the increased labor demand from the new MNEs causes foreign households to work even more. Like a mirror image, domestic households will work even less. Thus in this model, outsourcing helps bringing the economy back to the efficient long run equilibrium at a faster pace, but pushes aggregate employment further away from its efficient level in the initial periods. Specifically, the welfare effects are asymmetrical, as one country enjoys more leisure and the other works more. The model thus predicts that an unexpected depreciation attracts more FDI inflow, but could hurt domestic households as a result.

To assess how FDI affects welfare, we compute the expected world utility by numerically simulate the model in second order approximation. We compare the utility in the baseline model, in which free cross-border relocation is allowed, with utilities in two alternative scenarios: when all firms produce domestically, and when only long-term relocations are allowed. Our key findings include three observations. First, firm relocations in the steady state are welfare-improving, but short run FDI fluctuations further deteriorate welfare loss caused by nominal frictions through the aforementioned employment effects. Second, when monetary volatility gets sufficiently large, the short run loss outweighs the long run gain, and the world would be better off shutting down FDI completely. In our baseline calibration, FDI turns from being benecial to harmful at a standard deviation of the quarterly money supply shocks of 3.2%. Finally, the optimal strategy is to allow the long term rm relocation but discourage the short run, temporary relocations. We show that world utility is highest at every volatility level when the distribution of MNEs is kept constant at the steady state level, but further moving back-and-forth is prohibited in the short run.

Thus an important conclusion of the model is that short-run FDI fluctuations are welfare-reducing. As these temporary changes in investment flows are generated by firms who aim to take advantage of the inefficient real wage gaps caused by nominal frictions, they magnify the adverse effects of nominal rigidities on labor supply. A natural and important question stems from this study: Can policies be designed and implemented to reduce the above-mentioned inefficiency? And if so, how?

We provide an answer to this question from a monetary policy perspective. We augment the model with standard New Keynesian interest rate rules, and incorporate productivity shocks to convert the model into a suitable tool to address the question. The modified model produces the same positive relationship between the changes in FDI flows and exchange rate under both nominal and real shocks. We then investigate the effects of output gap response in interest-setting. Contrary to the well-known conclusion in closed-economy models that output gap response is counter-productive when the natural level of output is unknown, we show that when multinationals are present, output gap response is beneficial for a wide range of environmental settings. We show that targeting gross domestic product is a way to discourage fluctuations of FDI flow around its long-run equilibrium.

Previous literature has commonly seen output gap targeting as undesirable. In reality, the natural level of output optimally fluctuates in response to real shocks. As it is difficult for policymaker to separate changes caused by real shocks from those by nominal shocks, targeting output gap could mistakenly hamper the necessary adjustment of output to its natural level. But when multinationals exist, we argue that output gap response has an additional benefit. When firms move abroad to exploit a lower real wage caused by nominal rigidities, they relocation magnify welfare loss by further distorting the aggregate employments of both the source and recipient countries. As a country's GDP is directly related to its aggregate employment, having an interest rate rule that takes into account the output gap is a way to effectively responding to the temporary firm relocation over the business cycle.

We verify this conjecture by showing that targeting output gap significantly reduces the changes in FDI flows upon a nominal shock. Given this benefit, it is important to ask then, does this benefit of output gap response overcome the cost emphasized by previous literature?

To answer this question, we add aggregate productivity shocks to the model to generate deviations of the natural level of output. The model is then simulated in second order approximation to compute how the expected world utility change when we increase the output gap response coefficient. We find that the net benefit of output gap response is increasing with the relative volatility of nominal shocks compared to real shocks. Furthermore, we show that the cost of output gap response is proportional to the response coefficient. As a result, the cost is negligible when the coefficient is small. However, the marginal effectiveness of output gap targeting in the presence of nominal shocks is largest when the coefficient is small. Therefore, it is never optimal to have zero output response.

The rest of the paper is organized as follows. We give a brief review of the previous empirical and theoretical literature in section 2. In section 3, we set up the households' and firms' problems and describe the structure and environment of the model. We examine the long run equilibrium of the economy by studying the non-stochastic steady state in section 4. Section 5 studies the first order dynamics of the model. We first present a special case in analytical solutions to gain insights into the model, and then solve the model using numerical simulation to further study the details in the general case. Section 6 examines welfare effects of firm relocation, and demonstrate that short run fluctuations in FDI flows reduces world welfare. Section 7 incorporates interest rate rule and productivity shocks into the model to convert it into a suitable tool for monetary policy analysis. Section 8 analyze dynamics of the modified model. Section 9 shows that output response in an interest rule improves welfare. Section 10 concludes.

2 Review of previous literature

Theories explaining the determinants of FDI are abundant. Compared with traditional theories focusing on competitive advantages, such as better technology or managerial skill, studies examining the link between exchange rate and FDI are relatively new. Conventional wisdom suggests that a depreciation of domestic currency attracts FDI because it makes domestic goods cheaper, but earlier economists typically reject the idea. As when making an investment decision, what matters is the rate of return of the underlying asset. If, for example, a depreciation of the dollar makes a US asset cheaper to a potential foreign investor, one should also realize that the future benefits to be generated by the asset are also affected by the depreciation. In particular, if we believe that movements in exchange rate resemble a random walk, then in expectation, the value of future benefits of the asset must reduce by the same proportion as the cost of the asset. Therefore, as the depreciation leaves the rate of return unchanged, there is no reason that a depreciation should enhance foreigner's incentive to purchase US asset.

Despite the theoretical challenge, empirical works that started to emerge during the 1980s have consistently found a positive relationship between the exchange rate, defined as domestic currency price of foreign currency, and FDI inflows. We give a brief review of the empirical literature on the issue in the next subsection, and then a review of the theoretical literature following that.

2.1 The empirical literature

A large volume of empirical works relates the exchange rate and FDI flows. They provide overwhelming supports for the observed positive relationship between the two. These empirical works have led Blonigen (2005) to conclude in his survey paper on the empirical literature concerning the determinants of FDI flows that the level of exchange rate has consistently been found to be a significant determinant of short run FDI flows. In this section we give a brief review of some of these works. The purpose is not to write a comprehensive survey, but rather to point out some representative works that cover a variety of situations.

Cushman (1985, 1988) conducts the earliest empirical examinations of the relationship between the exchange rate and FDI flows. The two works use annual FDI flows between the U.S. and various developed countries. It is found that a host country currency appreciation decreases FDI inflows into the country. Subsequent empirical literature repeatedly confirms this finding. These works include Ray (1988), Froot and Stein (1991), Klein and Rosengren (1994), Grosse and Trevino (1996), Buch and Kleinert (2008). It should be noted that these works generally use FDI data between the U.S. and other developed countries, and one should expect that a large part of these FDI flows to represent horizontal, rather than vertical, type of FDI.

Starting from the late 1990s, a series of empirical works has cast their attention to the co-movement of exchange rate and FDI flows from developed countries to developing countries. Bayoumi and Lipworth (1998) examines Japanese FDI to various trading partners, and is one of the early works that uses data covering FDI to developing countries. Benassy-Quere et al.(2001) uses a rather comprehensive panel data set of

FDI flow from 17 OECD countries to 42 developing countries from 1984 to 1996. Other works examining Japanese outward FDI to developing countries include Kiyota and Urata (2004) and Xing (2006). Despite the difference in data and methodology, these works all confirm that a depreciation of the host country currency tends to attract more FDI inflows.

2.2 The theoretical literature

Froot and Stein (1991) is an early attempt to explain the relationship between exchange rate and FDI based on a relative wealth argument. The paper presents a theory of asset market imperfection that makes the cost of internal funds cheaper than borrowing externally. As a result, a depreciation increases foreign bidder's chance of success in acquiring a domestic asset, as it increases the relative value of the foreign bidder's wealth.

Blonigen (1997) presents a theory based on firm-specific asset. These are typically intangible assets such as patent or copyright. The model assumes that a foreign buyer's target is not the production facility, but rather the firm-specific asset, and foreign company sells their final products in their own country. Thus the model is able to break the chain between cost and benefit of the asset. For example, when a Japanese company buys a US firm, it could have acquired a production technology that enables it to produce and sell in Japan. In this manner, a dollar depreciation only lowers the cost of the acquisition in dollars, but leaves the benefits in yen unchanged. For this channel to work, some degree of goods market segregation is necessary, as otherwise, a US firm could likewise acquire the technology, produce, and sell the final products in Japan.

While these earlier theories provide insights on how to theoretically connect exchange rate and FDI, they also take a partial equilibrium perspective in examining the problem. In these models, exchange rate change is an exogenous and independent event. That is to say, the authors examine how a firm's incentive and profitability of acquiring a foreign asset would change, taken the change in exchange rate as given. Goldberg and Kolstad (1995) provide an early critique of the partial equilibrium approach by showing that when examining the effects of exchange rate, it is important to take into consideration how the demand of a firm's product co-vary with the exchange rate.

Aizenman(1992) and Russ(2007) write down general equilibrium models to examine the relationship between FDI flows and the volatility of exchange rate. Although being silent on the effects of the change in the level of the exchange rate, these models are direct predecessors of the current model. In Aizenman (1992), FDI is modeled as location choice made by potential MNE in a two period model. In order to produce in the second period, the firm has to make a choice in the first period regarding whether to establish one production facility or produce in both countries. A fixed cost of investment must be pre-committed the period before production takes place. With sticky wages, Aizenman shows that diversifying production location is a way for the firm to diversify risk, and thus concludes that a more volatile exchange rate encourages FDI flows. Russ(2007) incorporates Melitz(2003) type heterogeneous firms into an open-economy macro model developed by Devereux and Engel (2001). The work aims to examine horizontal type of FDI. Thus international trade is shut down, and firms are required to relocate to a foreign country in order to sell to the foreign market. Russ (2007) formalizes Goldberg and Kolstad(1994)'s idea that the covariance between demand and exchange rate matters, and argues that a more volatile exchange rate could encourage or discourage FDI investment depending on in which country the nominal shock originated.

3 The Model

There are two countries, denoted Home and Foreign, each inhabited by a continuum of households whose total mass is normalized to 1. That is, the world population is 2. In addition, there is a continuum of firms whose total mass is also 2. We assume that half of these firms are owned by Home households and the other half by Foreign households. Each firm is a monopolistic supplier of its differentiated good. The nature of these firms will be specified below.

3.1 Households' problem

A Home household h chooses consumption, labor supply, and money balance to maximize a time-separable utility function given by

$$U_t = E_t \sum_{j=0}^{\infty} \beta^j U_{t+j} \left(C_{t+j}(h), N_{t+j}(h), \frac{M_{t+j}(h)}{P_{t+j}} \right).$$

The period utility function at time t, which depends on consumption of a composite good $C_t(h)$, the labor supply $N_t(h)$, and real money balance $M_t(h)/P_t$ of household h, is given by

$$U_t = \frac{1}{1-\sigma} C_t(h)^{1-\sigma} - \frac{1}{1+\phi} N_t(h)^{1+\phi} + \chi \ln \frac{M_t(h)}{P_t}.$$

The composite good is a standard Dixit-Stiglitz CES aggregate of differentiated individual consumption goods defined as

$$C_t(h) = \left[\int_0^2 C_t(h,f)^{\frac{\mu-1}{\mu}} df \right]^{\frac{\mu}{\mu-1}},$$

where $C_t(h,f)$ is household h's consumption of the differentiated good produced by firm f. We assume that the elasticity of substitution μ is greater than 1. Each Home household h chooses its consumption basket taking the set of individual goods' prices $P_t(f)$ as given. Intra-temporal utility maximization gives us the familiar consumption price index P_t, defined as the minimum cost of acquiring one unit of the composite consumption good:

$$P_t = \left[\int_0^2 P_t(f)^{1-\mu} df \right]^{\frac{1}{1-\mu}},$$

while the Home country's demand for an individual product f is given by

$$C_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\mu} C_t, \tag{1}$$

where C_t is the aggregate Home consumption[4].

We follow Erceg, Henderson, and Levin (2000)'s approach to incorporate a Calvo type wage-setting mechanism. We assume that households are monopolistic suppliers of labor. Each household supplies a differentiated type of labor to the firms. Firms produce final products using the "composite labor", a CES aggregate of differentiated types of labor defined as

$$L_t = \left[\int_0^1 N_t(h)^{\frac{1}{1+\eta}} dh \right]^{1+\eta}.$$

Labor are assumed to be immobile across countries, and thus Home households supply labor only to firms who produce in the Home country, which include domestically-producing Home firms, and possibly Foreign firms who have relocated to produce in the Home country as multinationals. Foreign households act likewise. This implies the Home "wage index", which is defined as the minimum cost of hiring one unit of the composite Home labor, is given by

$$W_t = \left[\int_0^1 W_t(h)^{-\frac{1}{\eta}} dh \right]^{-\eta}, \tag{2}$$

where $W_t(h)$ is the wage rate set by household h for its differentiated type of labor. Like any individual firm, each household h has zero weight in the continuum of household, and therefore its particular wage rate

[4] That is, C_t is the sum of individual households' consumptions $C_t(h)$. We introduce complete asset market below, under which idiosyncratic consumption risks are eliminated, making $C_t(h) = C_t \; \forall h$.

has no effects on the aggregate wage. Thus any individual household sets a wage taking the aggregate wage rate as given. Like the monopolistically competitive firms, each household faces a downward-sloping labor demand curve for its particular type of labor given by

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\frac{1+\eta}{\eta}} L_t, \tag{3}$$

where L_t is the aggregate employment in the home country in period t.

Households engage in Calvo type wage setting. In each period t, with probability $(1-\theta)$, household h is able to update the wage rate it offers. Otherwise, its wage rate will be $\Pi W_{t-1}(h)$, where Π is the unconditional long-run gross rate of inflation of the economy, and $W_{t-1}(h)$ is the wage rate household h charged last period. Thus the wage rate is indexed to the long run level of aggregate inflation.

There is an integrated world financial market where a complete set of state-contingent nominal bonds are traded. These bonds are (arbitrarily) denominated in the Home currency. Thus, a Home household h faces the recursive period budget constraint

$$\begin{aligned} & P_t C_t(h) + \sum_{\nabla^{t+1} \in \Omega^{t+1}} Z\left(\nabla^{t+1}|\nabla^t\right) D\left(h, \nabla^{t+1}\right) + M_t(h) \\ \leq \quad & (1+\tau) W_t(h) N_t(h) + \Gamma_t(h) + T_t(h) + M_{t-1}(h) + D\left(h, \nabla^t\right), \end{aligned} \tag{4}$$

where $D\left(h, \nabla^{t+1}\right)$ is units of nominal bond household h acquires. Each of these bond pays one unit of Home currency in period $t+1$ in state ∇^{t+1}. The period t price of such a bond is denoted $Z\left(\nabla^{t+1}|\nabla^t\right)$. The set of all possible states in period $t+1$ is denoted Ω^{t+1}. $\Gamma_t(h)$ is the profit earned by Home-owned firms which was distributed to household h, $T_t(h)$ is government transfer, and τ is the rate of government subsidy to labor.

As Ricardian equivalence holds, we can assume that the government has a balanced budget every period without loss of generality. Thus the total government transfer (or tax when it is negative) to domestic households equals to the seigniorage revenue minus the expenditure on labor subsidy:

$$T_t = M_t - M_{t-1} - \tau W_t L_t.$$

We assume that the log of the money supply process follows a random walk

$$\frac{M_t}{M_{t-1}} = (1+\psi)\, e^{\upsilon_t}, \upsilon_t \sim N\left(0, \sigma_m^2\right).$$

The money supply shocks ν_t in this equation is a convenient way to capture disturbances to the nominal sectors of the economy, and is not intended to be taken literally as a shock created by the monetary authority. Rather, we can think of it as an error of monetary policy when the central bank is following a rule to increase money supply at a constant rate. The errors might be there because the actual money supply is difficult to measure precisely. The setting is also isomorphic to one in which money demand experiences exogenous shocks, e.g. when a credit crunch creates a drop in the velocity of money.

Household h's first order conditions from the utility-maximization problem include

$$Z\left(\nabla^{t+1}|\nabla^t\right) = prob\left(\nabla^{t+1}|\nabla^t\right) \beta \frac{\left[C_t(h)\right]^\sigma P_t}{\left[C_{t+1}(\nabla^{t+1}, h)\right]^\sigma P_{t+1}}, \tag{5}$$

which, by summing across all $t+1$ states, gives us the standard Euler equation

$$Z_t = \beta E_t \left[\frac{C_t^\sigma P_t}{C_{t+1}^\sigma P_{t+1}}\right],$$

where Z_t is the price of the riskless nominal portfolio that pays one unit of Home currency in every state at time $t+1$, i.e. the inverse of the nominal interest rate. Note that the existence of complete market allows us to drop the household index in consumption. The first order condition regarding money demand is thus

$$\frac{M_t}{P_t} = \left(\frac{1}{1-Z_t}\right) \chi C_t^\sigma,$$

where C_t and M_t are now interpreted as the average Home household consumption and money demand.

We assume that the government chooses the subsidy rate $\tau = \eta$ to correct the monopolistic distortion caused by the market power of wage-setters. The first order condition regarding the optimal wage-setting is then

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left[V_{N(h),t+j} - \frac{W_t(h)\Pi^j}{P_{t+j}} U_{c,t+j} \right] N_{t+j}(h) = 0. \tag{6}$$

where $V_{N(h),t+j}$ is the marginal disutility of labor for household h in period $t+j$, and $U_{c,t+j}$ is the marginal utility of consumption. When a household gets the chance to update its wage, it takes into account the fact that with probability θ every period, its current reset wage will remain effective into the future. Thus it optimally sets a wage by weighing the discounted sum of future marginal disutility of working against the marginal utility of consumption made possible by the extra income from working. As the individual labor demand function (3) makes clear, household h's reset wage at period t depends only on the current and expected future aggregate variables, but not on its household-specific history of wage-setting. Thus all wage-resetters in period t must set the same wage rate $\tilde{W}_t$. Given that a constant fraction $(1-\theta)$ of household reset wage in every period, the aggregate wage rate evolves according to

$$W_t = \left[(1-\theta)\, \tilde{W}_t^{-\frac{1}{\eta}} + \theta\, (\Pi W_{t-1})^{-\frac{1}{\eta}} \right]^{-\eta}. \tag{7}$$

Foreign households' optimization problem is symmetric to Home households', and we denote Foreign variables with an asterisk. Specifically, since the nominal bonds are denominated in the Home currency, the Foreign consumption Euler equation would be

$$Z_t = \beta E_t \left[\frac{C_t^{*\sigma} P_t^*}{C_{t+1}^{*\sigma} P_{t+1}^*} \left(\frac{S_t}{S_{t+1}} \right) \right],$$

where S_t is the nominal exchange rate, defined as the Home currency price of Foreign currency.

3.2 Firms' Problem

Each firm produces a differentiated product using a technology that is linear in the input of the "composite labor"

$$Y_t(\varphi) = \varphi L_t(\varphi), \tag{8}$$

where φ is the firm-specific productivity, and $Y_t(\varphi)$ and $L_t(\varphi)$ denote the output and labor hired by firm φ, respectively. (As firms only differ in terms of their productivity levels, we index firms by their productivities φ from now on.)

Firm heterogeneity in productivity is introduced mainly to capture the empirical regularity that multinationals are found to be the more productive firms in their industry, who earn higher revenue, hire more workers, and make larger profit. For example, Helpman, Melitz, and Yeaple (2004) examines the characteristics of multinational firms compared with exporters, as well as firms who only serve the domestic market. The paper concludes that among these three sets of firms, multinationals have the highest average productivity, followed by exporters and then domestic firms.

We assume that firms have heterogeneous productivity levels, but instead of having the firms paying a fixed cost to draw a random productivity, we assume that each Home firm has a randomly drawn, time-invariant productivity $\varphi \in \Phi_H$, while each foreign firm has a productivity $\varphi \in \Phi_F$. As in Helpman, Melitz, and Yeaple (2004) and Chaney (2008), productivities are assumed to be drawn from a Pareto distribution with shape parameter α. That is, the cumulative distribution function is given by

$$\Pr(\varphi' < \varphi) \equiv G(\varphi) = 1 - \left(\frac{\varphi_m}{\varphi} \right)^{\alpha}, \varphi \geq \varphi_m,$$

where φ_m is the lower bound of Home productivity distribution. The shape parameter α is an inverse measure of the heterogeneity of firms. A lower value of α indicates a fatter upper tail of the productivity distribution. We assume that $\alpha > \mu - 1$ to ensure that the $(\mu-1)^{\text{th}}$ moment of the distribution is bounded[5]. We also assume that the Home productivity distribution has a minimum value φ_m normalized to 1, but Foreign has a minimum value $\varphi_m^* < 1$. Thus Home firms have, on average, a higher productivity than that of Foreign.[6]

The vertical type FDI we meant to study is modeled as firms' production location decisions. To produce in period t, a firm has to first decide, in period $t-1$, where to produce next period. A firm can choose to produce either in its domestic country or in the foreign country. If it decides to move its production facility oversea, it has to pre-commit a fixed real cost F in terms of the aggregate consumption good. Producing at home requires no additional cost. We assume that the location decision is irreversible by time t, when production takes place. This decision-making is repeated every period. That is to say, instead of letting the firm pay a lump-sum sunk cost to establish an overseas facility and produce for multiple periods, we assume that the sunk cost is incurred every period. This sunk cost could represent advertisement cost, property taxes or administrative fees, etc. The assumption of a periodically repeated location decision is made mainly for tractability, but it should not change the qualitative results of the model. As will become clear later, the mechanism that induces short-term relocation of firms is the real wage gap caused by the sticky wage. In a Calvo setting, the wage rate adjusts sluggishly. If we assume that the firms' location decision is relevant for multiple periods, the decision rule will involve comparing a string of discounted future benefits with the sunk cost of relocation. While being more complicated, this alternative formulation does not change the qualitative nature of the decision.

Firms are assumed to flexibly set prices of their output. We also assume that goods are freely traded with no cost. Given this assumption, the law of one price holds for each individual good. Consider the location decision of a Home firm with productivity φ. If it decides to produce in the Home country in period t, its expected discounted profit is given by

$$E_{t-1}\left\{\delta_t\left[P_t(\varphi)\,Y_t(\varphi) - W_t L_t(\varphi)\right]\right\}. \tag{9}$$

If it produces in the Foreign country instead, the expected discounted profit is

$$E_{t-1}\left\{\delta_t\left[P_t(\varphi)\,Y_t(\varphi) - S_t W_t^* L_t^*(\varphi)\right]\right\}.$$

In these expressions, $\delta_t \equiv \beta P_{t-1} C_{t-1}^{\sigma} / P_t C_t^{\sigma}$ is the standard stochastic discount factor. Law of one price ensures that the firm receives the same unit price on its sales in both countries, when expressed in a common currency. The firm's total units sold worldwide is denoted $Y_t(\varphi)$.

Combined with the production function (8) and demand function (1), the firm's profit maximization problem at time t when producing domestically can be written

$$\max_{P_t(\varphi)} P_t(\varphi)\,Y_t(\varphi) - W_t \frac{Y_t(\varphi)}{\varphi}$$

subjected to the demand function

$$Y_t(\varphi) = \left(\frac{P_t(\varphi)}{P_t}\right)^{-\mu} Y_t^w, \tag{10}$$

where $Y_t^w \equiv C_t + C_t^* + m_t F$ is the total world demand for the composite good. Note that as the fixed cost is assumed to be in units of the composite consumption goods, period t world demand is increased by the amount $m_t F$ when cross-border relocations present. We denote the total mass of MNEs in period t by m_{t-1}.

[5]This is necessary because otherwise, the total revenue made by MNEs will be unbounded when the cutoff productivity goes to infinity. The point will be made clear below.

[6]An alternative assumption is to assume that the distribution of Home firms has a different shape parameter α than that of Foreign firms. All qualitative results we present below only require that Home firms are more productive on average and do not depend on how the difference in average productivity is introduced.

Since consumption utility functions are identical across countries, purchasing power parity (PPP) holds for the aggregate prices: $P_t = S_t P_t^*$. Therefore a firm's relative prices in the two countries must be equal. This fact enables us to abbreviate the individual demand into (10). Profit-maximization implies the standard optimal price-setting as a constant markup over the marginal cost under monopolistic competition:

$$P_t(\varphi) = \frac{\mu}{\mu - 1}\left(\frac{W_t}{\varphi}\right). \tag{11}$$

Substituting the optimal price and output into the profit function (9), the expected discounted operational profit when the firm produces in the domestic country is

$$\left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu} E_{t-1}\left\{\delta_t \left(\frac{W_t}{\varphi}\right)^{1-\mu} P_t^{\mu} Y_t^{w}\right\}.$$

Likewise, we can show that the expected discounted operational profit of firm φ when producing as a multinational is

$$\left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu} E_{t-1}\left\{\delta_t \left(\frac{S_t W_t^*}{\varphi}\right)^{1-\mu} P_t^{\mu} Y_t^{w}\right\}.$$

The firm's location decision then depends on whether the gain from moving production oversea is large enough to compensate for the sunk cost. The firm will become a multinational iff

$$\left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu} E_{t-1}\left\{\delta_t \left[\left(\frac{W_t}{\varphi}\right)^{1-\mu} - \left(\frac{S_t W_t^*}{\varphi}\right)^{1-\mu}\right] P_t^{\mu} Y_t^{w}\right\} \geq S_{t-1} P_{t-1}^* F.$$

Substitute the expression of the stochastic discount factor into the equation above, use the PPP condition, and rearrange, we get

$$\varphi^{\mu-1}\beta\left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu} E_{t-1}\left\{\frac{u_{c,t}}{u_{c,t-1}}\left[\left(\frac{P_t^*}{W_t^*}\right)^{\mu-1} - \left(\frac{P_t}{W_t}\right)^{\mu-1}\right] Y_t^{w}\right\} \geq F. \tag{12}$$

The left-hand-side is the real gain from switching production abroad, and the right-hand-side is the real cost. This relationship suggests that, a lower real wage in the foreign country, and therefore a lower marginal cost of production, is the source of potential gain from going multinational. Specifically, the lower is the foreign real wage compared to domestic real wage, the larger is the gain. Moreover, the scope of the gain is a monotonically increasing function of the firm's productivity level, implying that there exists a cutoff productivity $\tilde{\varphi}_t$ such that in period t, a Home firm with productivity $\varphi = \tilde{\varphi}_t$ will just break even in expectation by switching production location to the Foreign country in period $t+1$, and every Home firm with productivity level $\varphi > \tilde{\varphi}_t$ must expect to make a positive net discounted profit by relocating and thus must decide to produce as a multinational in the next period.[7]

3.3 Aggregation

The labor demand in the Home country is given by an integration of individual firm's labor demand over all firms producing in the Home country. Suppose in period t, some Home firms are producing in the Foreign country as multinationals[8]. Using the production function (8), the demand for individual goods (1), and

[7] We follow the convention to index variable by the time its value is determined. Thus the cutoff productivity $\tilde{\varphi}_t$ determines which firms are producing abroad in period $t+1$.

[8] The case of reverse FDI flow is symmetric to the current analysis. And we show later that given the assumption that Home firms have higher average productivity, FDI flow from Home to Foreign is the relevant situation to look at.

the optimal price-setting (11), we can write the aggregate labor demand as a function that depends only on aggregate variables and a measure of aggregate productivity in the Home market:

$$\begin{aligned} L_t &= \int_1^{\tilde{\varphi}_{t-1}} \left(\frac{Y_t(\varphi)}{\varphi}\right) g(\varphi)\, d\varphi \\ &= \int_1^{\tilde{\varphi}_{t-1}} \left(\frac{P_t(\varphi)}{P_t}\right)^{-\mu} Y_t^w \varphi^{-1} g(\varphi)\, d\varphi \\ &= \left(\frac{\mu}{\mu-1}\right)^{-\mu} \left(\frac{W_t}{P_t}\right)^{-\mu} Y_t^w \bar{\varphi}_{Ht-1}^{\mu-1}, \end{aligned} \tag{13}$$

where

$$\bar{\varphi}_{Ht-1} \equiv \left[\int_1^{\tilde{\varphi}_{t-1}} \varphi^{\mu-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\mu-1}} \tag{14}$$

is a measure of the aggregate productivity of firms who operate in the Home market. Likewise, the labor demand in the Foreign market is given by

$$\begin{aligned} L_t^* &= \int_{\tilde{\varphi}_{t-1}}^{\infty} \left(\frac{Y_t(\varphi)}{\varphi}\right) g(\varphi)\, d\varphi + \int_{\varphi_m^*}^{\infty} \left(\frac{Y_t^*(\varphi^*)}{\varphi^*}\right) g(\varphi^*)\, d\varphi \\ &= \left(\frac{\mu}{\mu-1}\right)^{-\mu} \left(\frac{W_t^*}{P_t^*}\right)^{-\mu} Y_t^w \left[\bar{\varphi}_{Ht-1}^{*\mu-1} + \bar{\varphi}^{*\mu-1}\right], \end{aligned} \tag{15}$$

where

$$\bar{\varphi}_{Ht-1}^* \equiv \left[\int_{\tilde{\varphi}_{t-1}}^{\infty} \varphi^{\mu-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\mu-1}} \tag{16}$$

is the aggregate productivity of Home firms who produce in the Foreign market in period t, i.e., the multinationals, and

$$\bar{\varphi}^* \equiv \left[\int_{\varphi_m^*}^{\infty} \varphi^{*\mu-1} g(\varphi^*)\, d\varphi^*\right]^{\frac{1}{\mu-1}}$$

is the mean productivity of the Foreign firms. Meanwhile, integrating the individual budget constraint (4) across household and using the government budget constraint, we get a country budget constraint

$$P_t C_t + \sum_{\nabla^{t+1} \in \Omega^{t+1}} Z\left(\nabla^{t+1} | \nabla^t\right) D\left(\nabla^{t+1}\right) = L_t W_t + \Gamma_t + D\left(\nabla^t\right).$$

Variables in this expression are interpreted as household average values. Specifically, Γ_t denotes the total profit earned by Home-owned firms, which include Home firms who produce domestically, and Home firms who produce in the Foreign country. In case multinational firms present, these profits is reduced by the amount of the fixed sunk cost

$$\Gamma_t = \Gamma_{Ht} + \Gamma_{Ht}^* - (1 - G(\tilde{\varphi}_t)) P_t F,$$

where the subscript H is a reminder of the origin of the firm. Using the optimal price-setting rule (11), for a Home firm φ who produces domestically, its profit is

$$\begin{aligned} \Gamma_{Ht}(\varphi) &= P_{Ht}(\varphi) Y_{Ht}(\varphi) - W_t L_{Ht}(\varphi) \\ &= \left(\frac{1}{\mu-1}\right) W_t L_{Ht}(\varphi) \end{aligned}$$

Integrating over domestically-producing Home firms using a similar procedure as in (13), we find that the total profit made by domestically-producing Home firms is given by, in real terms

$$\frac{\Gamma_{Ht}}{P_t} = \left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu}\left(\frac{W_t}{P_t}\right)^{1-\mu} Y_t^w \bar{\varphi}_{Ht-1}^{\mu-1}. \tag{17}$$

Likewise, the total real profit made by the set of Home firms producing as MNEs is

$$\frac{\Gamma_{Ht}^*}{P_t} = \left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu}\left(\frac{W_t^*}{P_t^*}\right)^{1-\mu} Y_t^w \bar{\varphi}_{Ht-1}^{*\mu-1}. \tag{18}$$

In the next section, we will use these aggregate profits to determine steady state consumption.

4 Linearized Model: A Study of the Steady State

We will proceed by linearizing the model around a non-stochastic steady state. In the steady state, all real variables have constant values and all nominal variables grow at a constant rate Π. We denote the value of a variable at steady state by an overbar.

In the non-stochastic steady state, every household sets the same wage rate and supply the same amount of labor, implying $\bar{L} = \bar{N}(h)$ and $\bar{W} = \bar{W}(h)$ across every household. The wage distribution degenerates into a single value. The optimal wage-setting rule (6) reduces to the simple familiar labor supply rule that equates the marginal rate of substitution and the real wage

$$\bar{\omega} = \bar{L}^{\phi}\bar{C}^{\sigma}, \tag{19}$$

$$\bar{\omega}^* = \bar{L}^{*\phi}\bar{C}^{*\sigma}, \tag{20}$$

where $\bar{\omega} \equiv \bar{W}_t/\bar{P}_t$ denotes the real wage. Dividing the home labor supply by its foreign counterpart, we get

$$\frac{\bar{\omega}}{\bar{\omega}^*} = \left(\frac{\bar{L}}{\bar{L}^*}\right)^{\phi}\left(\frac{\bar{C}}{\bar{C}^*}\right)^{\sigma}. \tag{21}$$

Incorporating aggregate labor demands (13) and (15) into (21), labor market equilibrium in the steady state thus requires

$$\left(\frac{\bar{\omega}}{\bar{\omega}^*}\right)^{\frac{1}{\phi}+\mu} = \left(\frac{\bar{C}}{\bar{C}^*}\right)^{\frac{\sigma}{\phi}}\left(\frac{\overline{\bar{\varphi}}_{Ht-1}^{\mu-1}}{\overline{\bar{\varphi}}_{Ht-1}^{*\mu-1} + \bar{\varphi}^{*\mu-1}}\right), \tag{22}$$

where $\overline{\bar{\varphi}}_H$ and $\overline{\bar{\varphi}}_H^*$ are the steady state values of the average productivities defined in (14) and (16).

The budget constraint states that in steady state, each country's consumption is the sum of its residents' labor income and the net profits earned by domestic firms. Thus for the Home country,

$$\bar{C} = \bar{\omega}\bar{L} + \bar{\Gamma}_t/\bar{P}_t.$$

And $\bar{\Gamma}_t$, the total profits made by Home firms, is the sum of profits made by domestic producers and that made by multinationals. Taking the steady state value of (13), (17), and (18) and substitute into the above expression, we find that the steady state consumptions is given by

$$\bar{C} = \left(\frac{\mu}{\mu-1}\right)^{1-\mu}\bar{\omega}^{1-\mu}\bar{Y}^w\overline{\bar{\varphi}}_H^{\mu-1} + \left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu}\bar{\omega}^{*1-\mu}\bar{Y}^w\overline{\bar{\varphi}}_H^{*\mu-1} - \left(1 - G\left(\tilde{\varphi}\right)\right)F. \tag{23}$$

Similarly, the foreign consumption is given by

$$\bar{C}^* = \left(\frac{\mu}{\mu-1}\right)^{-\mu}\bar{\omega}^{*1-\mu}\bar{Y}^w\overline{\bar{\varphi}}_H^{*\mu-1} + \left(\frac{\mu}{\mu-1}\right)^{1-\mu}\bar{\omega}^{*1-\mu}\bar{Y}^w\bar{\varphi}^{*\mu-1}. \tag{24}$$

The steady state level of FDI flow cannot be solved explicitly, but we could prove that in the steady state, there are positive FDI flows and its direction depends on the average productivities of the countries.

Proposition: When the countries have different average productivities and are otherwise identical, in the steady state, multinational firms originating from the high-productivity country relocate to produce in the low-productivity country.

Proof: see appendix

Following Melitz(2003) and subsequent literature, we interpret the fixed cost F as the embodiment of barrier to cross-border FDI flows. When F goes to infinity, it becomes excessively expensive to relocate oversea and cross-border direct investment shuts down, leaving the economy in a domestic production equilibrium. In such an equilibrium, labors in each country work for domestic firms, and consumption equals to the total revenue of the firms. When F shrinks to zero, there is no barrier to direct investment, and the break-even condition (12) states that FDI flows will only stop when real wages in the two countries are equalized. As we cannot obtain a closed form solution to the equilibrium cutoff productivity, we next study these two limiting cases to obtain a better understanding of the steady state equilibrium.

It should be noted that in Melitz (2003), zero fixed trade cost induces a corner solution in which all firms become exporters. As there are always positive demand for their products oversea and as monopolist, the firms always make a positive profit by serving these demand. In this model though, even if the fixed cost drops to zero, an interior solution is still guaranteed because real wages adjust to equilibrate the marginal cost of production in equilibrium, and at that point, there is no extra gain to be made by relocation and the economy attains an equilibrium distribution of MNEs.

As the break-even condition makes clear, the fixed cost also serves as a filter that selects the most productive firms to be MNEs. When a finite fixed cost exists, a firm has to be productive enough to earn a profit at least as large as the sunk cost to switch to a foreign location. Thus the set of multinational firms are the upper truncation of the productivity distribution. When there is no fixed cost at all, we don't exactly know who among the Home firms become multinationals. But here we consider the special case $F = 0$ to be the limit of the equilibria when we shrinks F. Thus we impose the requirement that the most productive firms are the first to become MNEs.

In a domestic production equilibrium, as the average productivities in the two markets are exogenous, we can solve explicitly the consumption ratio. As the labor market equilibrium condition reduces to

$$(\bar{\omega}^r)^{\frac{1}{\phi}+\mu} (\bar{C}^r)^{-\frac{\sigma}{\phi}} = \left(\frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}}\right),$$

where $x^r = x/x^*$ denotes the relative Home-Foreign value of the variable x. And the ratio of the budget constraints obtains

$$\bar{C}^r = (\bar{\omega}^r)^{1-\mu} \left(\frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}}\right).$$

We can thus solve for the relative real wage, consumption, and employment as

$$\bar{\omega}^r = \left(\frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}}\right)^{\frac{\phi+\sigma}{1+\phi\mu+\sigma(\mu-1)}} > 1,$$

$$\bar{C}^r = \left(\frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}}\right)^{\frac{1+\phi}{1+\phi\mu+\sigma(\mu-1)}} > 1,$$

and

$$\bar{L}^r = \frac{\left(\frac{\bar{W}_t}{\bar{P}_t}\right)^{-\mu} \bar{\varphi}^{\mu-1}}{\left(\frac{\bar{W}_t^*}{\bar{P}_t^*}\right)^{-\mu} \bar{\varphi}^{*\mu-1}} = (\bar{\omega}^r)^{-\mu} \left(\frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}}\right) = \left(\frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}}\right)^{\frac{1-\sigma}{1+\phi\mu+\sigma(\mu-1)}}$$

which could be greater than or less than 1 depending on the value of σ.

When the fixed cost F shrinks to zero, real wages must equalize in equilibrium. Using the identity

$$\bar{\varphi}^{\mu-1} = \overline{\varphi}_H^{\mu-1} + \overline{\varphi}_H^{*\mu-1},$$

The labor market equilibrium implies

$$\bar{C}^r = \left(\frac{\bar{\varphi}^{\mu-1} - \overline{\varphi}_H^{*\mu-1}}{\overline{\varphi}_H^{*\mu-1} + \bar{\varphi}^{*\mu-1}} \right)^{-\frac{\phi}{\sigma}}.$$

This tells us that consumption ratio is negatively correlated with the ratio of aggregate productivity in the two countries. That's because the relative aggregate productivity positively affects labor demand. When, for example, aggregate productivity is lowered at Home by Home firm's relocation, Home agents work less, decreasing their marginal disutility of working. Utility maximization then implies that the marginal utility of consumption must be relatively low for Home consumers, i.e. they must be consuming relatively more. When Home firms move abroad to produce, $\overline{\varphi}_H^{*\mu-1}$ increases, lowers relative Home productivity, and that requires Home relative consumption to increase.

On the other hand, the two budget constraints can be combined to get

$$\bar{C}^r = \frac{\mu\overline{\varphi}_H^{\mu-1} + \overline{\varphi}_H^{*\mu-1}}{(\mu-1)\,\overline{\varphi}_H^{*\mu-1} + \mu\bar{\varphi}^{*\mu-1}} = \frac{\mu\bar{\varphi}^{\mu-1} - (\mu-1)\,\overline{\varphi}_H^{*\mu-1}}{(\mu-1)\,\overline{\varphi}_H^{*\mu-1} + \mu\bar{\varphi}^{*\mu-1}}.$$

This tells us that the relative consumption decreases as Home firms move abroad, because they don't get the labor income by working for these MNEs anymore. As the aggregate productivities $\overline{\varphi}_H^{\mu-1}$ and $\overline{\varphi}_H^{*\mu-1}$ are functions of the cutoff productivity, these two equations pin down the steady state cutoff productivity independently of the other endogenous variables in the system.

Clearly, in the equilibrium, relative consumption is greater than one. Home is consuming a larger portion of world income. The reason is that, the multinationals are monopolistically competitive firms who make a positive profit, and they repatriate the profit to Home agents. If they had instead their entire revenue paid to their host country, whether in the form of labor income or dividends, then consumptions have to equalize, as the consumption ratio implied by the budget constraints in that situation must equal to 1 to satisfy the labor market equilibrium condition.

The equilibrium employment ratio is determined by the aggregate productivity ratio and thus

$$\bar{L}^r = \frac{\overline{\varphi}_H^{\mu-1}}{\overline{\varphi}_H^{*\mu-1} + \bar{\varphi}^{*\mu-1}} < 1.$$

It is fairly clear that consumption ratio must always stay above 1 as we vary the value of F, because of the effect of profit-repatriation on Home consumption. When F is positive, the profit made by MNEs will be reduced by the cost of fixed investment, but as the marginal MNE just breakeven, the set of MNEs as a whole must repatriate a positive net profit that is at least as large as the profit these firms would have made by operating at home. Thus the consumption ratio must still be greater than one. The fact that the Home residents consume a larger portion of total world output implies that the MNEs re-export the majority of their output, a fact that conforms to empirical findings.

5 The first-order dynamics of MNE activity and the exchange rate

In this section, we examine the dynamics of the model in a first order approximation. In the first subsection, we analytically solve for the first-order dynamics of firms' location decisions under a special assumption to gain insights into the model. In the next subsection, we present results from numerical analysis of the general model.

5.1 Analytical solutions for a special case

An analytically tractable solution for the level of FDI is possible when we approximate the model around a steady state where the fixed investment cost is zero. As we will see, despite being a special case, the analytical study in this section unveils the basic mechanism behind the first-order dynamics of the location distribution of firms, and thus provides the intuitive explanation of the positive correlation between the exchange rate and FDI inflows.

Although we approximate around a steady state in which the fixed cost is zero, we still include the fixed cost in our first order approximation, as this enables us to have a say about how the magnitude of the fixed cost would affect the short run deviations of the variables of the system. The thought experiment here is to ask, starting from a steady state when the fixed cost is zero, how an infinitesimally small increase in the level of the fixed cost affect the endogenous variables when a shock hits.[9]

Denote the log deviation from steady state of a variable x by $\hat{x} \equiv \log(x/\bar{x})$. In a steady state when the fixed cost is zero, the real wages in the countries must equalize in equilibrium. The break-even condition (12) can be linearized into a simple form:

$$E_t \left\{ \hat{P}^*_{t+1} - \hat{W}^*_{t+1} - \hat{P}_{t+1} + \hat{W}_{t+1} \right\} = \left(\frac{\mu}{\mu - 1} \right)^{\mu} \left(\frac{\bar{\omega}}{\bar{\bar{\varphi}}} \right)^{\mu - 1} \frac{F}{\beta \bar{Y}^w}. \tag{25}$$

Note that the right hand side of this expression is the expected real wage gap between Home and Foreign in the non-stochastic steady state, and thus the break-even condition requires firm relocation to continue until the real wage gap is brought back to its long run level in expectation.

On the other hand, linearizing the optimal wage setting rule (6) and combining with the individual labor demand (3) and aggregate wage evolution (7), we yield an equation dictating the evolution of the aggregate wage[10]:

$$\hat{W}_t - \hat{W}_{t-1} = \frac{(1-\beta\theta)(1-\theta)}{\theta \left(1 + \phi \left(\frac{1+\eta}{\eta}\right)\right)} (mrs_t - \hat{\omega}_t) + \beta E_t \left(\hat{W}_{t+1} - \hat{W}_t \right), \tag{26}$$

where mrs_t is the deviation of the marginal rate of substitution between consumption and leisure and $\hat{\omega}_t$ is the real wage. The aggregate wage evolution closely resembles a New Keynesian Phillips Curve when the final goods prices are set a la Calvo. The current wage inflation depends partly on the current level of the real variables, and partly on the expected wage inflation next period. Specifically, when the marginal rate of substitution is high compared with the real wage in period t, an average household's disutility from working exceeds the utility-based rewards from the labor income, and the wage-resetters in the economy ask for a higher wage rate. A symmetric equation can be solved for the Foreign wage inflation. Substituting the marginal rate of substitution and the real wage into equation (26) above and rearrange, we get

$$(1 + \kappa + \beta) \hat{W}_t = \kappa \left(\phi \hat{L}_t + \sigma \hat{C}_t + \hat{P}_t \right) + \hat{W}_{t-1} + \beta E_t \hat{W}_{t+1}, \tag{27}$$

$$\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta \left(1 + \phi \left(\frac{1+\eta}{\eta}\right)\right)},$$

where $\hat{L}_t$ is the (deviation of) aggregate employment. Money market equilibrium relates price and consumption to the exogenous money supply shock $\sigma \hat{C}_t + \hat{P}_t = \hat{M}_t$. Substitute into the equation above and taking difference with its foreign counterpart, we get

$$(1 + \kappa + \beta) \hat{W}^d_t = \kappa \left(\phi \hat{L}^d_t + \hat{M}^d_t \right) + \hat{W}^d_{t-1} + \beta E_t \hat{W}^d_{t+1},$$

[9] A more consistent way to go is probably to leave F out of the first order system, but given that the first order system is linear, setting the fixed cost to zero everywhere in this section gives us the precise solution to the endogenous variables. Thus the inclusion of F does not cause any loss in precision.

[10] The derivation follows exactly that of Erceg, Henderson, and Levin (2000). See the appendix of that paper for the precise procedure.

where $\hat{x}^d \equiv \hat{x} - \hat{x}^*$ denotes the deviation of the relative Home-Foreign variable. The difference of aggregate employments given by (13) and (15) can be linearized to get

$$\hat{L}_t^d = -\mu \hat{W}_t^d + \mu \hat{M}_t^d + (\mu - 1)\,\widehat{\overline{\varphi}}_{Ht-1} - (\mu - 1)\left(\frac{\overline{\varphi}_H^{*(\mu-1)}}{\overline{\varphi}_H^{*(\mu-1)} + \bar{\varphi}^{*(\mu-1)}}\right)\widehat{\overline{\varphi}}_{Ht-1}^*. \tag{28}$$

Note that given the assumption that the productivity distributions are Pareto, the aggregate productivities (14) and (16) can be expressed as functions of the cutoff productivity $\tilde{\varphi}_{t-1}$. The aggregate productivity of domestically producing Home firms is

$$\bar{\varphi}_{Ht-1}^{\mu-1} = \frac{\alpha}{\alpha - (\mu - 1)}\left(1 - \tilde{\varphi}_{t-1}^{\mu-1-\alpha}\right),$$

which implies the first order relationship

$$\widehat{\overline{\varphi}}_{Ht-1} = \frac{\alpha}{(\mu - 1)}\left(\frac{\overline{\tilde{\varphi}}}{\overline{\overline{\varphi}}_H}\right)^{\mu-1}\overline{\tilde{\varphi}}^{-\alpha}\widehat{\tilde{\varphi}}_{t-1}. \tag{29}$$

Similarly, we can show that the aggregate productivity of the set of MNEs is a decreasing function of the cutoff productivity:

$$\widehat{\overline{\varphi}}_{Ht-1}^* = -\frac{\alpha}{(\mu - 1)}\left(\frac{\overline{\tilde{\varphi}}}{\overline{\overline{\varphi}}_H^*}\right)^{\mu-1}\overline{\tilde{\varphi}}^{-\alpha}\widehat{\tilde{\varphi}}_{t-1}. \tag{30}$$

Substituting (29), (30) and (28) into (27) and rearrange, we get

$$(1 + \kappa + \beta + \kappa\varphi\mu)\,\hat{W}_t^d = (\kappa\phi\mu + \kappa)\,\hat{M}_t + z\widehat{\tilde{\varphi}}_{t-1} + \hat{W}_{t-1}^d + \beta E_t \hat{W}_{t+1}^d \tag{31}$$

where

$$z = \kappa\phi\alpha\overline{\tilde{\varphi}}^{\mu-1-\alpha}\left(\frac{1}{\overline{\overline{\varphi}}_H^{\mu-1}} + \frac{1}{\overline{\overline{\varphi}}_H^{*(\mu-1)} + \bar{\varphi}^{*(\mu-1)}}\right).$$

Substituting (25) into (31), we can solve for the relative nominal wage rate as

$$\hat{W}_t^d = \alpha_1 \hat{M}_t^d + \alpha_2 \widehat{\tilde{\varphi}}_{t-1} + \alpha_3 \hat{W}_{t-1}^d + \alpha_4 \frac{F}{\overline{Y}w}, \tag{32}$$

where

$$\alpha_1 = \frac{(\kappa\phi\mu + \kappa + \beta)}{(1 + \kappa + \beta + \kappa\phi\mu)} \in (0, 1)$$

$$\alpha_2 = \frac{\kappa\phi\alpha\overline{\tilde{\varphi}}^{\mu-1-\alpha}}{(1 + \kappa + \beta + \kappa\phi\mu)}\left(\frac{1}{\overline{\overline{\varphi}}_H^{\mu-1}} + \frac{1}{\overline{\overline{\varphi}}_H^{*(\mu-1)} + \bar{\varphi}^{*(\mu-1)}}\right) > 0,$$

$$\alpha_3 = \frac{1}{(1 + \kappa + \beta + \kappa\phi\mu)} \in (0, 1)$$

$$\alpha_4 = \frac{\left(\frac{\mu}{\mu-1}\right)^{\mu}\left(\frac{\bar{\omega}}{\overline{\tilde{\varphi}}}\right)^{\mu-1}}{(1 + \kappa + \beta + \kappa\phi\mu)} > 0.$$

This solution has made clear that the difference in the aggregate wages, which is a key variable affecting firm's relocation decisions, only partially adjust to the relative monetary shocks. As in each period, only a portion $(1 - \theta)$ of households is able to adjust their wage rate. The relative wage depends positively on its own lagged value, and positively on the lagged cutoff productivity. As a higher cutoff productivity implies that fewer Home firms are producing abroad, and this drives up the relative Home wage through an increased labor demand. Meanwhile, an increment in the fixed cost also positively contributes to the enlargement of the wage gap, as the increased impediment to FDI flows retains some would-be MNE firms in their domestic country, and therefore increases the domestic labor demand.

Using the fact that the option to invest abroad restricts the expected difference in the real wages to a function of the constant fixed cost given in (25), we use the solution to the relative wage rate (32) to solve for the first order dynamics of the cutoff productivity as

$$\widehat{\tilde{\varphi}}_t = \beta_1 \hat{M}_t^d + \beta_2 \widehat{\tilde{\varphi}}_{t-1} + \beta_3 \hat{W}_{t-1}^d + \beta_4 \frac{F}{\bar{C}^w} \tag{33}$$

where

$$\beta_1 = \left(\frac{1}{(1+\kappa+\beta+\kappa\phi\mu)\left(\kappa\phi\alpha\bar{\tilde{\varphi}}^{\mu-1-\alpha}\right)} \right) \left(\frac{1}{\bar{\tilde{\varphi}}_H^{\mu-1}} + \frac{1}{\bar{\tilde{\varphi}}_H^{*(\mu-1)} + \bar{\varphi}^{*(\mu-1)}} \right)^{-1} > 0$$

$$\beta_2 = -\frac{1}{(1+\kappa+\beta+\kappa\phi\mu)}$$

$$\beta_3 = -\left(\frac{1}{(1+\kappa+\beta+\kappa\phi\mu)\left(\kappa\phi\alpha\bar{\tilde{\varphi}}^{\mu-1-\alpha}\right)} \right) \left(\frac{1}{\bar{\tilde{\varphi}}_H^{\mu-1}} + \frac{1}{\bar{\tilde{\varphi}}_H^{*(\mu-1)} + \bar{\varphi}^{*(\mu-1)}} \right)^{-1}$$

$$\begin{aligned} \beta_4 \quad = \quad & \left[\frac{\beta\left(1+\kappa+\beta+\kappa\phi\mu\right)^2 - (\kappa+\beta+\kappa\varphi\mu)}{(1+\kappa+\beta+\kappa\phi\mu)} \right] \frac{1}{\left(\kappa\phi\alpha\bar{\tilde{\varphi}}^{\mu-1-\alpha}\right)} \\ & \times \left(\frac{\mu}{\mu-1}\right)^{\mu} \left(\frac{\bar{\omega}}{\bar{\tilde{\varphi}}}\right)^{\mu-1} \left(\frac{1}{\bar{\tilde{\varphi}}_H^{\mu-1}} + \frac{1}{\bar{\tilde{\varphi}}_H^{*(\mu-1)} + \bar{\varphi}^{*(\mu-1)}} \right)^{-1}. \end{aligned}$$

Note that the fraction of Home firms who will be producing as multinationals in period $t+1$ is $1 - G(\tilde{\varphi}_t)$. Thus the cutoff productivity is negatively related to the size of FDI. The solution to the cutoff productivity (33) thus predicts that the size of FDI inflow into the Foreign country is negatively related to the relative money supply. For example, when in period t, Foreign has a monetary expansion relative to Home, it causes increased FDI flows from Home to Foreign as it decreases the cutoff productivity. Meanwhile, as PPP holds in this model, the exchange rate changes one-to-one with relative money supplies

$$\hat{S}_t = \hat{M}_t^d.$$

Take the example above, when Foreign has a monetary expansion relative to Home, Foreign currency depreciates relative to Home. Thus the exchange rate is positively correlated with the size of FDI. As the relative price of the consumption goods are held constant at one by the PPP condition, what attracts Home firms to produce in the Foreign country is a decrease in the labor cost. Given the solution to the relative nominal wage (32), it is straightforward to show that the Foreign real wage depreciates after a Foreign monetary expansion. An increase in the Foreign money supply causes a proportional nominal depreciation of the Foreign currency, and nominal rigidity in the labor market prevents the wages to fully adjust to compensate for the increased price abroad. Thus in real terms, this causes a depreciation of the Foreign real wage against that of Home, attracting the more productive Home firms to move productions oversea.

5.2 Relative Real Wage and FDI: a Numerical Analysis of the General Case

In this section, we numerically simulate the model for the general case when the fixed cost is positive, and highlight several key features.

The parameterization follows mainly Erceg, Henderson, and Levin (2000). We set a (quarterly) discount rate $\beta = 0.99$, and utility parameters $\sigma = \phi = 1.5$. Money demand parameter χ is set to one, as it only influences the levels of the steady state aggregate prices. We choose an elasticity of substitution $\mu = 4$, and wage mark-up rate $\eta = 1/3$. As of the probability for any individual household to having sticky wage, we set $\theta = 0.75$. This corresponds to an average wage contract duration of one year. Following Russ (2007), we set $\alpha = \mu + 0.1$ to make the dispersion of firm productivity close to one, as the cross-industry estimation by Helpman, Melitz, and Yeaple (2004) suggests[11]. The Foreign minimum productivity is set to 1/2, and this produces a steady state real wage ratio of 1.72. We set the fixed cost to 0.01, at which value about 5% Home firms produce as multinationals in the steady state.[12] The money supply shocks have a standard deviation of 0.02.

Figure 1 plots the impulse response functions of the cutoff productivity $\tilde{\varphi}_t$ and the relative real wage $\omega_t^r \equiv (W_t/P_t)/(W_t^*/P_t^*)$ to a shock of one standard deviation to the Foreign money supply ν^*. Note that unlike in the special case when the fixed cost is zero, when we have a positive fixed cost, the relative real wage does not jump back to its long-run level over merely one period. This is because when the fixed cost is positive, the Home real wage is higher than the Foreign real wage in the steady state. As a result, a linearization of the break-even condition yields

$$\begin{aligned} & \left(\bar{\omega}^{*-(\mu-1)} - \bar{\omega}^{-(\mu-1)}\right) \left[(\mu-1)\hat{\varphi}_t + E_t\hat{U}_{c,t+1} - \hat{U}_{c,t} + E_t\hat{C}^w_{t+1}\right] \\ & +\bar{\omega}^{*-(\mu-1)}(\mu-1)E_t\left[\hat{P}^*_{t+1} - \hat{W}^*_{t+1}\right] - \bar{\omega}^{-(\mu-1)}(\mu-1)E_t\left[\hat{P}_{t+1} - \hat{W}_{t+1}\right] \\ = & \ 0. \end{aligned} \tag{34}$$

Compared with the break-even condition in the last section when $F = 0$, in the general case, relocation of firms causes the expected real wage gap between the two countries to shrink, but not to an extent as to completely eliminate the deviation of the relative real wage from its long run value. Specifically, the current and expected deviations of consumption play a role in this adjustment process, and the relative real wage returns to its long run level sluggishly. It is clear from Figure 1 that this gradual appreciation of the Foreign real wage against that of Home corresponds to reduced FDI flows from Home to Foreign. As when the relative real wage returns to its long run level, the extra profitable opportunity diminishes, causing some of the less productive Home firms among the MNEs to move production facilities back to the Home country each period. Notably, the appreciation of the Foreign real wage and the returning of the temporary Home MNE firms to domestic production happen during about the same length of time. As we should expect, the length of time necessary for the relative real wage to converge back to its long run level is positively affected by the degree of wage stickiness θ.

[11] Helpman, Melitz, and Yeaple (2004) estimated productivity dispersion, measured by the value of the coefficient $\alpha - (\mu - 1)$, using industry level data for U.S. and European firms. The estimations of these industry-specific dispersion parameters range from 0.52 to 2.97. The qualitative simulation results in this section are robust to these alternative dispersion parameters. See Helpman, Melitz, and Yeaple (2003), NBER working paper#9439, Table A.1.

[12] We consider these reasonable values given that the two countries in the model represent a developed-developing country pair. We try different parameter values to make sure the qualitative results are robust to alternative parameterizations.

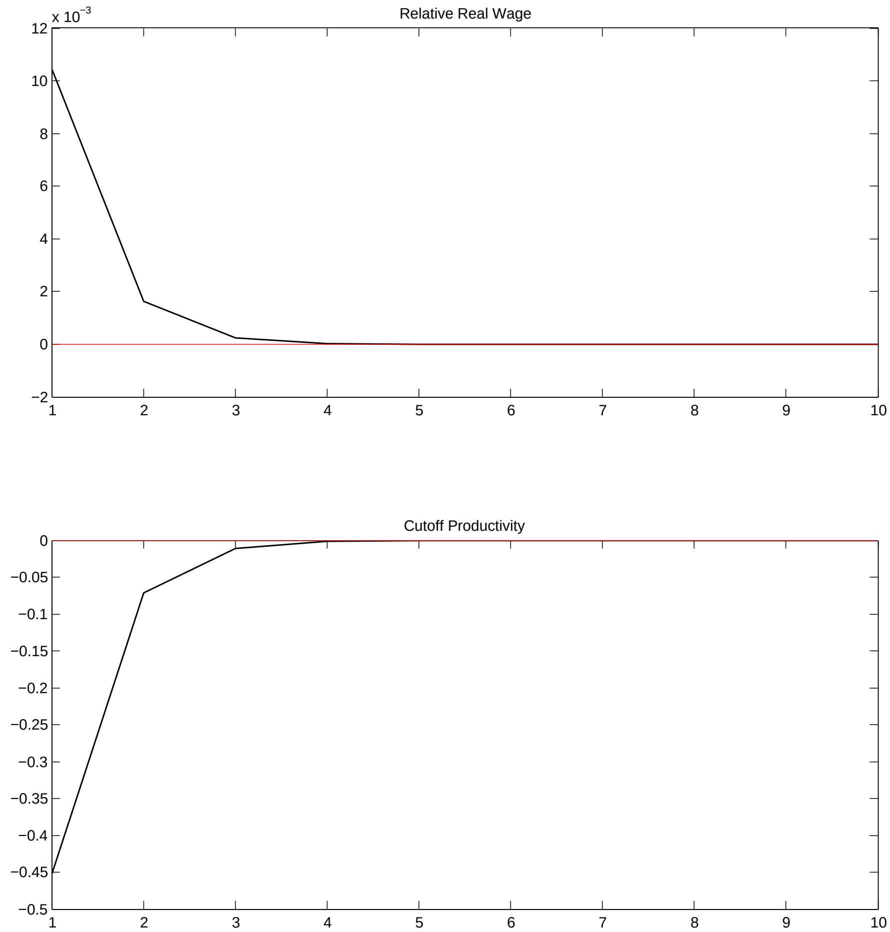


FIGURE 1: IRF, Orthogonalized Shock to Foreign Money Supply

While the relative real wage takes more than one period to converge back to its long run level in the general case, it remains true that allowing cross-border firm relocation significantly speeds up the adjustment procedure of the labor market back to its long run equilibrium after a shock hits. Figure 2 reports the impulse response functions of consumptions, employments, the nominal wages and prices under a shock to the Foreign money supply. Figure 3 plots the impulse response functions of these variables under a domestic production scenario, i.e., when cross-border relocation of firms is not allowed. While all variables smoothly converge under domestic production, allowing FDI brings the aggregate employment levels as well as real wages to their long run equilibrium level at a much faster rate. The kinks in the impulse responses are caused by the assumption that the actual production cannot take place until one period after the shock.

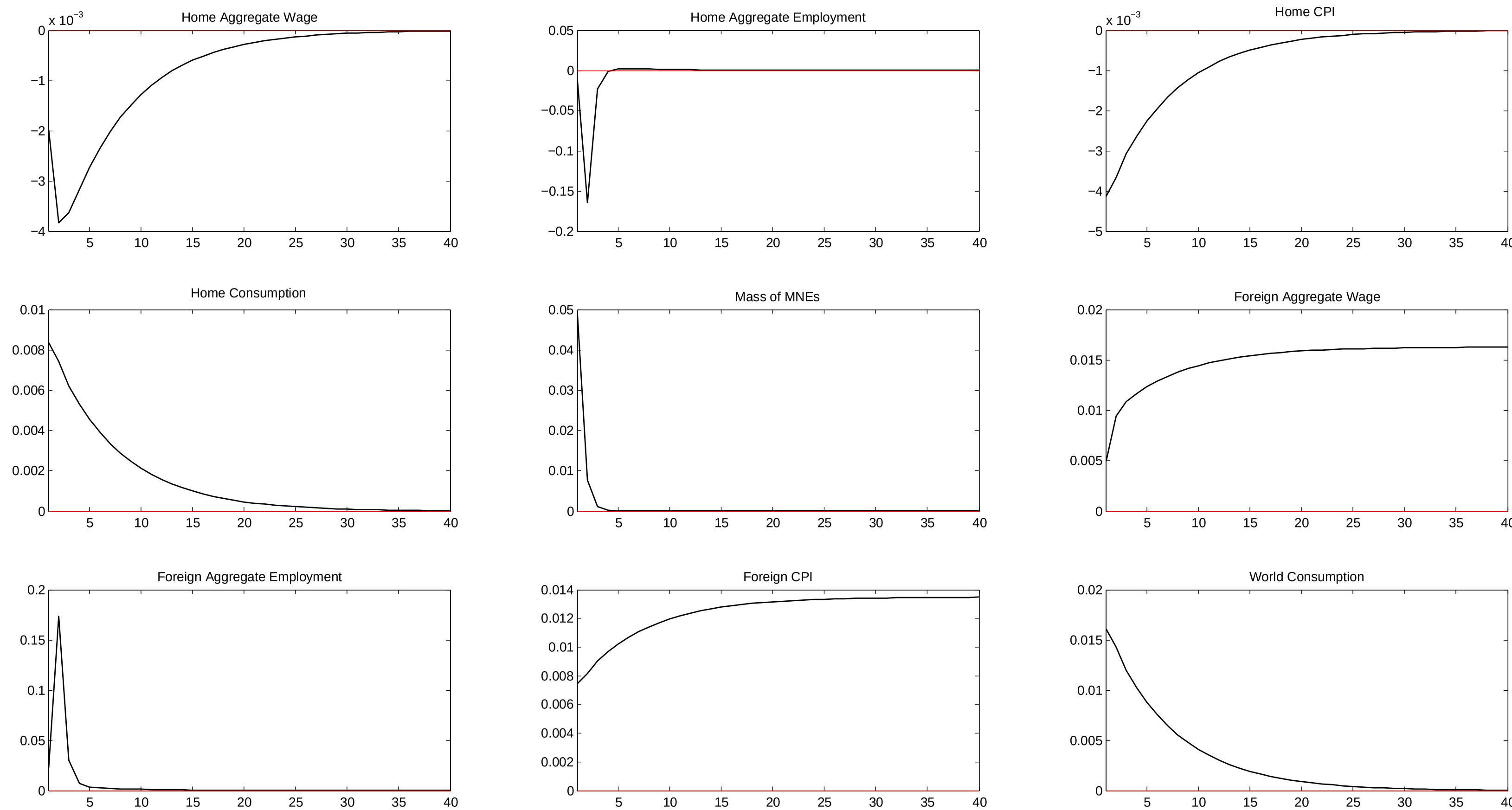


FIGURE 2: IRF, Orthogonalized Shock to Foreign Money Supply

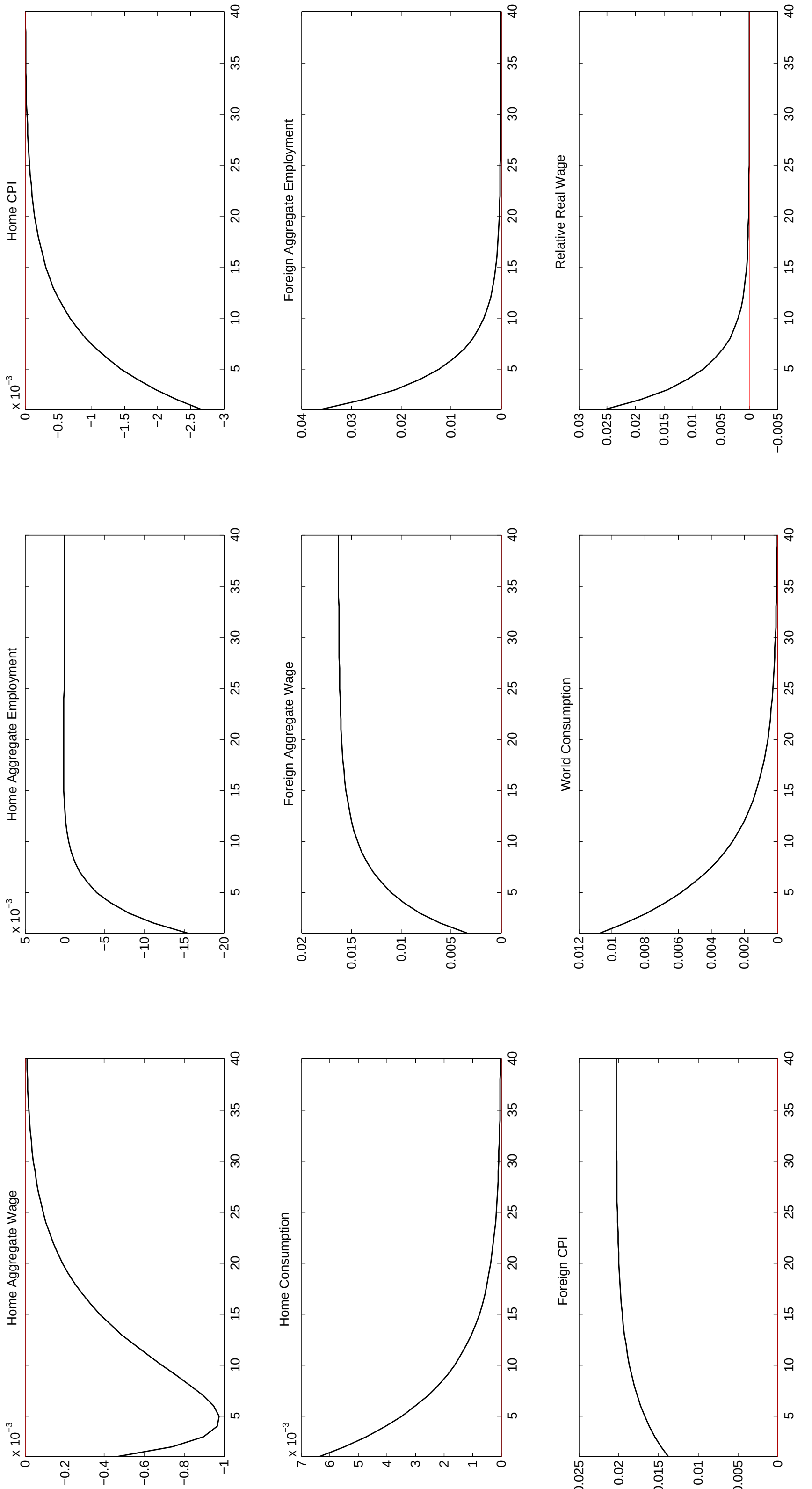


FIGURE 3: IRF under Domestic Production

The option of production relocation puts an upper cap on the gap between the real wages, as cross-border FDI flow enables the firms to take advantage of a lower real wage abroad, until the gap in the real wages shrinks to a point such that the marginal firm cannot gain any additional net benefit by relocation. The effectiveness of this mechanism partly depends on the magnitude of the fixed cost. When the fixed cost is zero, cross-border production relocation is going to equalize the real wages, and therefore bring the relative real wage back to its long run equilibrium, within a single period.

Although the option of production relocation brings the economy back to its long run equilibrium at a faster rate, it does so at the cost of exacerbating the distortions to aggregate employments. Figure 2 depicts the dynamics of aggregate employments following the shock. The sharp drop (rise) of the Home (Foreign) aggregate employment one period after the shock is caused by the relocation of the new MNEs. The more productive Home firms are attracted by the lower real wage abroad after the shock, and their relocation means the demand for Foreign labor is further increased. Without these new MNEs, Foreign (Home) workers would already be working too much (little) compared to the flexible-price long run equilibrium level, as the Foreign monetary expansion lowers (increases) the Foreign (Home) real wage. The entry of new MNEs thus drives the aggregate employments in both country further away from their long run equilibrium level, at least in the initial periods.

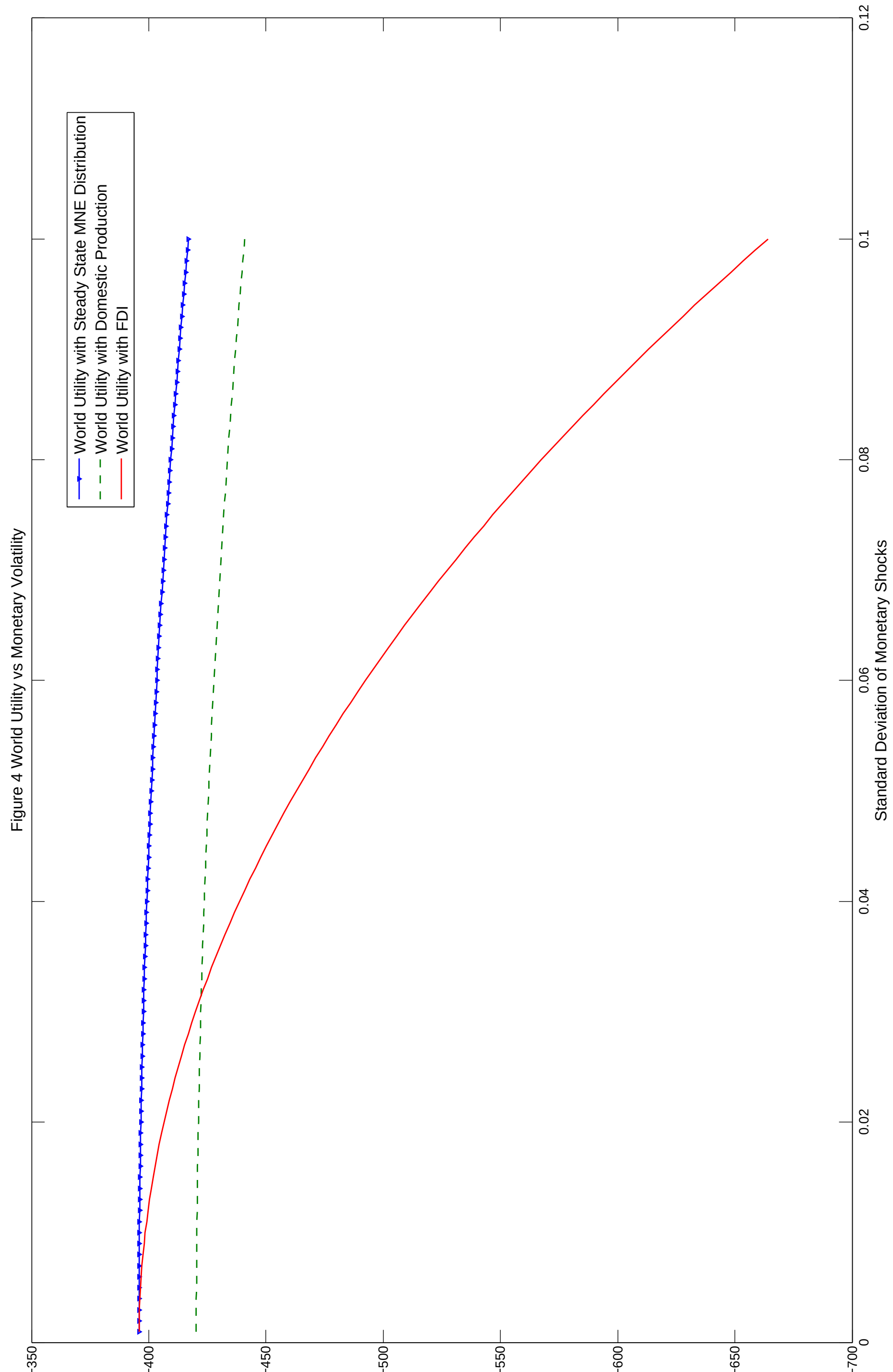
Note that as complete market distributes consumptions consistently over time, the change in relative utility of the two countries is mainly determined by the change in employment levels. This suggests that the welfare effects of these cross-border relocations after a specific shock is asymmetric across the two countries. As the Foreign real wage depreciation attracts more FDI inflows, Foreign households are worse off as they end up working more, while households in the Home country enjoy more leisure. Thus the model suggests that an unanticipated depreciation attracts more FDI inflows, but leaves the domestic agents worse off as they are forced to work more under sticky wage contracts. In this setting, it is not beneficial to the domestic household to manipulate the exchange rate to attract FDI inflow. Rather, the reversed operation of an artificial appreciation that drives firms abroad is welfare-improving. Of course, the gain is only to the appreciating country, the world as a whole loses utility from such nominal rigidity-caused deviations.

6 Welfare Effects of FDI

To formally analyze how the option of cross-border relocation affects expected utility, in this section we numerically simulate the model in second order approximation. We compare the total world utility in three scenarios: when FDI is allowed, when all firms produce in their domestic country, and when the steady state MNE distribution is enforced. We show that while the steady state world utility is higher when FDI is allowed, the utility decreases at a faster rate as the volatility of monetary shocks increases. As a result, when the volatility of monetary shocks gets large enough, the world will be better off if cross-border FDI flows are completely shut down. Yet given that the long run relocation of firms in the steady state is welfare-improving, we argue that if a capital control mechanism can be enforced so as to hamper the short run fluctuations of FDI, but allow the long-term investors to freely relocate, world utility would be strictly higher than under either free FDI flows or under complete domestic production.

We define the world utility function as the sum of the Home and Foreign utilities, as both countries have equal population. To compute the expected world utility, we simulate the model for two thousands periods, and drop the first two hundred. We repeat this for various values of monetary volatilities, measured by the standard deviations of the shocks to money growth ν and ν^*. We compute the expected utility under these different values of standard deviations, ranging from 0.1% to 10%, at increment of 0.1%.

Figure 4 reports the results of these simulations. The solid red line depicts the world utility computed from the baseline model, when all firms can freely relocate cross-border. The dotted green line is the world utility under domestic production, i.e., when all firms are restricted to produce in their domestic country. Clearly, the steady state FDI flows are welfare-improving. As the countries are assumed to be endowed with different levels of productivity, the high productivity Home firms relocate to Foreign to seek lower production cost, benefiting both countries as it balances the marginal rate of substitutions. Thus world utility in the baseline model exceeds the utility in the domestic production scenario when the volatility of monetary shocks is zero, and remains higher when the volatility is small.



However, while utilities under both the FDI and domestic production scenarios decrease with the volatility of monetary shocks, the rate of utility decrement is higher with FDI flows. This is because when the volatility gets higher, on average, the relative real wage is driven further away from its flexible-price equilibrium level. This then induces more temporary relocation of firms, causing aggregate employments to deviate even more, as we have shown in the last section. Thus the option to relocate in the short run amplifies the inefficiency caused by the nominal rigidity in wage-setting. As a result, if the monetary volatility is sufficiently high, it becomes welfare-improving to shut down cross-border FDI flows altogether. In our baseline calibration, the utility with FDI drops below the utility under domestic production at a standard deviation of 3.2%, which is large considering these are quarterly simulation, but still not impossibly large.

These welfare results can be compared with various applications of the theory of second best. When a friction causes market failure, allowing market participants to freely exploit the opportunities caused by the friction could deteriorate the overall welfare loss, calling for optimal policy intervention. Yet in our specific case, the same FDI flow motivated by lower production cost abroad could cause good or harm depending on the underlying cause of that lower real wage in the first place. It is welfare-improving when the real wage gap is caused by the fundamental difference in productivity, but exacerbates the welfare loss when the wage gap is caused by short run nominal friction. Therefore, although we have shown that at large enough volatility, agents will be better off in domestic production equilibrium, the optimal policy prescription should target those firm relocations of a short run, temporary nature. The top blue line with triangular markers in Figure 4 depicts the world utility when we simply keep the steady state distribution of firms, but disallow further moving back-and-forth. World utility is highest under this scenario at every volatility level, as the economy is able to reap the gain from long run firm relocation, but avoid the harm caused by short run relocations aimed to take advantage of nominal rigidities.

The interesting question then, is how such a policy could be implemented when the "steady state distribution" is unknown, or when firm productivity is not directly observable. One possibility is to require a minimum operation time, for example, disallowing sale of principle capital in three years. One might also impose a capital tax to discourage the temporary relocations. In either case, the key is to make entry profitable for the long-term investors, but deter the short-term profit seekers. Designing such a policy would be interesting future research.

7 Interest Rate Rule and Productivity Shocks

7.1 Interest Rate Rule and Output Gap Response

The basic setup of the model is the same as in chapter one. Instead incorporating money demand into the utility function, we now close the model with explicit interest rate rules. Assume that monetary authorities in both countries set nominal interest rates according to a Taylor type rule. The Home country's central bank set nominal interest rate according to

$$i_t = \rho + \rho_\pi (\pi_t - \bar{\pi}) + \rho_y \hat{Y}_t + \nu_t, \tag{35}$$

where ρ denotes the steady state net nominal interest rate, which equals to $\frac{1}{1-\beta} - 1$ in a zero-inflation steady state, and ν_t is a shock to the nominal interest rate. The Foreign country's nominal interest is set in a symmetric manner. Assume that the shocks to the nominal interest rates are normally distributed.

Note that in equation (35), $\hat{Y}_t$ denotes the output gap, defined as the percentage deviation of current output from the *steady state* output. Theoretically, it is optimal for monetary policymakers to target instead the deviation of output from its natural level - the output that would result in a frictionless environment. But in practice, it is very difficult to disentangle observed economic fluctuations into nominal shocks versus real shocks-induced parts, and thus the natural level of output is almost always unknown. As a result, it is more practical to instead respond to the output deviation from the steady state (or balanced growth path) value.

Likewise, monetary authority should adjust the nominal interest rate to its natural level, implying the constant term ρ in equation (35) should optimally be the time-varying natural rate. Thus, the nominal shock ν_t can be seen as a policy error due to the fact that the natural rate is unobservable.

Also, it is noteworthy that the output measure we use is the gross domestic product (GDP), which measures total product in a given period within a nation's geographical border. When multinationals are present, a country's total GDP counts products produced by Foreign-owned firms. Thus when a Home firm moves to Foreign to produce, its output is no longer counted in the Home GDP, but is included in the Foreign GDP.

Following the convention, Home GDP is defined as the aggregate product of all firms that produce in the Home country. The aggregation takes the same form as consumption:

$$Y_t = \left[\int_1^{\tilde{\varphi}_{t-1}} \left(Y_{Ht}(\varphi)\right)^{\frac{\mu-1}{\mu}} g(\varphi)\, d\varphi\right]^{\frac{\mu}{\mu-1}}. \tag{36}$$

Likewise, for Foreign country, GDP includes the products of those Home firms that produce in the Foreign country as multinationals:

$$Y_t^* = \left[\int_{\varphi_m^*}^{\infty} \left(Y_t^*(\varphi)\right)^{\frac{\mu-1}{\mu}} g(\varphi)\, d\varphi + \int_{\tilde{\varphi}_{t-1}}^{\infty} \left(Y_{Ht}^*(\varphi)\right)^{\frac{\mu-1}{\mu}} g(\varphi)\, d\varphi\right]^{\frac{\mu}{\mu-1}}. \tag{37}$$

Incorporating interest rate rules does not change the basic properties of the model. In fact, our baseline model in chapter one with money demand is equivalent to a model with an interest rate rule in the form of (35) targeting the consumption gap instead of output gap. Yet there are two important reasons to use a rule with output gap response: First, it is an integral part of the widely used Taylor rule. Second, as we will show in the next section, aggregate output measured by GDP is closely associated with the aggregate employment in each country. Given our earlier finding that short-run FDI fluctuations magnify the welfare losses by adversely affecting aggregate employments, the output gap response is a way to respond to the "employment gap". Thus we expect output gap targeting to be an effective tool to control the short run fluctuations in FDI flows.

Under interest rate rules, FDI flows and exchange rate still exhibit the expected positive correlation. We verify that a stronger output gap response coefficient ρ_y is welfare-improving under the shocks to the nominal interest rate.[13]

However, to conclude that output response improves welfare in an environment with only nominal shocks leaves out an important aspect of the optimal monetary policy discussion. As is well known, nominal shocks do not shift the natural level of output - the output level that would result with flexible prices - over business cycle. Therefore, under current assumptions, the deviation of output from its steady state level is equivalent to its deviation from the natural level. Yet when the natural level of output does deviate from its steady state level, there is an important cost responding to output gap. The cost raises from the fact that in practice, the natural level of output is unobservable. As a result, although monetary policy should only be used to correct for the deviation of output from its natural level, in reality it is more practical to target instead the deviation from the steady state or balanced growth path. Under this type of constraints, when a real shock causes a deviation of the natural level of output, the output gap response will be inefficient as it acts as another disturbance. To see this, note that our definition of the output gap can be decomposed into two parts: a first component that represents the deviation of output from its natural level, and a second component that represents the deviation of the natural level of output from the steady state level. I.e.

$$\hat{Y}_t \equiv \log\left(Y_t/\bar{Y}\right) = \log\left(\frac{Y_t}{Y_t^n} \times \frac{Y_t^n}{\bar{Y}}\right) = \log\left(\frac{Y_t}{Y_t^n}\right) + \log\left(\frac{Y_t^n}{\bar{Y}}\right),$$

where Y_t^n denotes the natural level of output. Note that nominal rigidities cause the first term to deviate from zero, which should be the concern of monetary policy. A real shock causes the natural level of output

[13] We leave the graphs to the appendix.

to optimally deviate from steady state, in which case the second term will be non-zero. Yet when monetary authorities can't separate the second term from the first, the interest rate rule (35) can be written as

$$i_t = \rho + \rho_\pi (\pi_t - \bar{\pi}) + \rho_y \log\left(\frac{Y_t}{Y_t^n}\right) + \rho_y \log\left(\frac{Y_t^n}{\bar{Y}}\right) + \nu_t. \tag{38}$$

In this case, the "overcompensation" represented by the term $\rho_y \log\left(Y_t^n/\bar{Y}\right)$ looks like an augmentation to the nominal interest shock ν_t, and it increases the economic fluctuations in a manner as such. It is easy to infer from this expression that the inefficiency caused by mistakenly responding to the natural level deviation is increasing in both the magnitudes of the real shocks and the response coefficient ρ_y.

Based on this argument, the literature has in general favored inflation targeting over output targeting. (See, e.g. Gali (2008)) However, when multinationals present, there is an additional benefit of output gap response. As we have shown in chapter one, the short run changes in FDI flows cause further distortions in the labor market, which ultimately cause the welfare loss we highlighted. When a firm changes its production location in response to a nominal shock, it causes a corresponding change in labor demand and output of its host country. In that sense, when monetary policy takes into account the output gap, it automatically corrects for the short run changes in employment, and thus discourages those relocation of temporary nature.

This gain, however, should be weighed against the cost the existing literature has emphasized. Therefore, we will need to incorporate a real shock into the model to do a meaningful investigation of the pros and cons of output gap response.

7.2 Productivity Shocks and Firm Price-setting

Next, we add aggregate productivity shocks to the model. This type of real shocks are commonly used in the real business cycle literature, and are easily interpretable in the context of previous studies of monetary policy.

Assume each country gets an aggregate, country-specific productivity shock. Firms in this model have heterogeneous productivity, but we assume that the aggregate productivity shock affects all firms in a country in completely symmetric ways. Specifically, we assume that the production function of a Home firm with productivity φ is given by

$$Y_t(\varphi) = \varphi A_t L_t(\varphi),$$

where A_t represents the aggregate Home productivity shock. Foreign firms have the symmetric production functions with Foreign aggregate productivity shock denoted by A_t^*.

We abstract from idiosyncratic firm productivity shocks. Although it might be more realistic, it adds relatively little to our current analysis. Idiosyncratic shocks change firms' relative position in the productivity distribution, but eventually the volume of cross-border FDI flow is determined by the relative aggregate productivity of Home and Foreign.

Another issue here is that the aggregate productivity is firm-augmenting, rather than labor-augmenting. In models without multinationals, these are equivalent. But when firms are allowed to move across border, it becomes an important distinction. In fact, they could have very different implications on the relationship between FDI flows and exchange rate. For example, if Foreign labors become more productive, it could cause a Foreign appreciation together with an FDI inflow, as firms seek high-productivity workers abroad, and this result would contradict the empirical finding we summarized in chapter one. We think in our context, firm-augmenting technology is more realistic and reasonable because, when considering cross-border FDI flows, we typically see firms to be the patent holder. For a firm, the adoption of a new technology or product line is a more likely motivation for FDI than more productive foreign labors. We consider this especially true in the short run, the time horizon of our major interest.

We assume the productivity shocks follow the processes

$$\log A_t = \zeta \log A_{t-1} + \epsilon_t,$$

$$\log A_t^* = \zeta \log A_{t-1}^* + \epsilon_t^*,$$

in which ϵ_t and ϵ_t^* are normally distributed.

Assume also that like the workers in the baseline model, firms are optimally subsidized at the rate $\tau_p = 1/(\mu - 1)$. Thus for every unit of final product a firm sells to consumers, the firm receives $(1+\tau_p)p_t(\varphi)$. This assumption is made to ensure the optimality of the nonstochastic steady state.

We assume that each country's government subsidizes its own firms, even if the firm is producing abroad. Government subsidization to foreign firms to attract FDI is by itself an interesting topic, but it is not our main focus in this paper.

As the result of the optimal subsidy, firms produce the optimal output, at which prices equal to their respective marginal costs. Thus a Home firm producing at Home will optimally set price at

$$p_t(\varphi) = \frac{W_t}{\varphi A_t}, \tag{39}$$

and for Home firms producing in Foreign, the optimal price reflects its Foreign labor cost and Home aggregate productivity:

$$P_{Ht}^*(\varphi) = \frac{W_t^*}{\varphi A_t}. \tag{40}$$

For Foreign firms, the optimal price is

$$P_t^*(\varphi^*) = \frac{W_t^*}{\varphi^* A_t^*}.$$

As in the baseline model, we assume that there is no trade cost, and therefore the law of one price holds for every single good, which then implies that the purchasing power parity holds.

7.3 Location Choice: The break-even condition

Now we turn to a typical firm's location choice problem. As in chapter one, we let firms choose production locations one period ahead of time, and if a firm decides to produce in the foreign country, it has to pre-commit a sunk cost F. When a Home firm with productivity φ produces at Home, it earns a profit

$$\Pi_t^H = \left(\frac{1}{\mu - 1}\right) W_t^{1-\mu} (\varphi A_t)^{\mu-1} P_t^{\mu} Y_t^w, \tag{41}$$

and if producing in the Foreign country, it earns a profit

$$\Pi_t^F = \left(\frac{1}{\mu - 1}\right) (S_t W_t^*)^{1-\mu} (\varphi A_t)^{\mu-1} P_t^{\mu} Y_t^w \tag{42}$$

in Home currency. It is easy to see that with the aggregate productivity shock, the firm's relative profit producing in Foreign country still only depends on the wage gap between the two countries, as it takes its technology with it wherever it produces. Like before, the firm will choose to produce as a multinational only if the discounted expected gain from moving abroad is large enough to compensate for the current fixed foreign investment cost F. I.e.

$$E_t\left[\delta_{t+1}(\Pi_{t+1}^F - \Pi_{t+1}^H)\right] \geq P_t F,$$

where $\delta_{t+1}$is the standard stochastic discount factor. Substitute the profit functions (41) and (42) into the expression above, use the optimal prices (39) and (40), and rearrange, we can reduce the inequality to

$$\left(\frac{\beta}{\mu - 1}\right) \varphi_t^{\mu-1} E_t \left\{ \frac{C_t^{\sigma}}{C_{t+1}^{\sigma}} (A_{t+1})^{\mu-1} Y_{t+1}^w \left[\left(\omega_{t+1}^*\right)^{1-\mu} - \left(\omega_{t+1}\right)^{1-\mu} \right] \right\} \geq F. \tag{43}$$

The break-even firm that is indifferent between producing at Home versus Foreign has a productivity$\tilde{\varphi}_t$ that exactly equalizes the two sides of the inequality (43). As the real gain from moving abroad (the left hand side of the inequality) is increasing in firm productivity φ, every firm with a higher productivity than the cutoff productivity chooses to be a multinational in the following period. Note that a higher expected aggregate productivity in the Home country makes Home firms more likely to be a multinational, as it enhances the marginal gain from accessing lower cost labors in the same manner as the firm-specific productivity.

7.4 Aggregation: Labor demand and GDP

Through aggregation across producing firms, Home and Foreign labor demands are given by, respectively,

$$L_t = \bar{\varphi}_{H,t-1}^{\mu-1} \omega_t^{-\mu} (A_t)^{\mu-1} Y_t^w,$$

and

$$L_t^* = \left[\left(A_t \bar{\varphi}_{H,t-1}^* \right)^{\mu-1} + \left(A_t^* \bar{\varphi}^* \right)^{\mu-1} \right] (\omega_t^*)^{-\mu} Y_t^w.$$

Substituting demand and optimal prices into the definition of country outputs (36) and (37), we can conveniently write each country's output as a function of its current period's aggregate employment and a measure of the aggregate productivity of firms who produce in that country:

$$Y_t = \bar{\varphi}_{Ht-1} A_t L_t,$$

and

$$Y_t^* = \left[\left(\bar{\varphi}_{Ht-1}^* A_t \right)^{\mu-1} + \left(\bar{\varphi}^* A_t^* \right)^{\mu-1} \right]^{\frac{1}{\mu-1}} L_t^*.$$

These expressions resemble a country-level aggregate productivity function in models with representative firms. The difference, however, is that the aggregate productivity of the recipient country is positively affected by the source country's aggregate productivity shock. Given a certain level of labor supply L_t^*, a positive Home productivity shock increases the Foreign GDP.

8 IRFs: the relationship between FDI flows and exchange rate

In this section, we simulate the model to examine the behavior of variables under nominal and real shocks. Our baseline case sets the inflation response coefficient and output response coefficient respectively to $\rho_\pi = 1.5$ and $\rho_y = 0$. The magnitude of real shocks are set to equal the magnitude of the nominal shocks in the baseline model to 0.02, although we will examine implications of different relative volatility in the next section. The persistence coefficient of the aggregate productivity shocks are set to $\zeta = 0.9$. Other parameters are identical to those of the baseline model in chapter one. Impulse response functions (IRFs) of the system in a first order approximation are plotted below. As the impulse responses are symmetric, we only show the IRFs after a shock to the Home country.

8.1 Nominal shock: IRFs after an orthogonalized shock to ν_t

Starting with a nominal shock. the figure below depicts IRFs of important variables after a one-standard-deviation orthogonalized shock to ν_t. This corresponds to an increase in the Home interest rate, or a Home monetary contraction. The impulse responses of real variables of the system are identical to that of a reduction in Home money supply in the previous chapter. Specifically, we still observe a positive relationship between the real exchange rate, measured by the relative real wage $\omega_t^r \equiv \omega_t / \omega_t^*$. As in chapter one, because the Calvo type wage-setting leads to sluggish adjustment, the Home real wage appreciation increases the expected payoff of producing abroad, prompting Home firms to seek cheap labor by relocating.

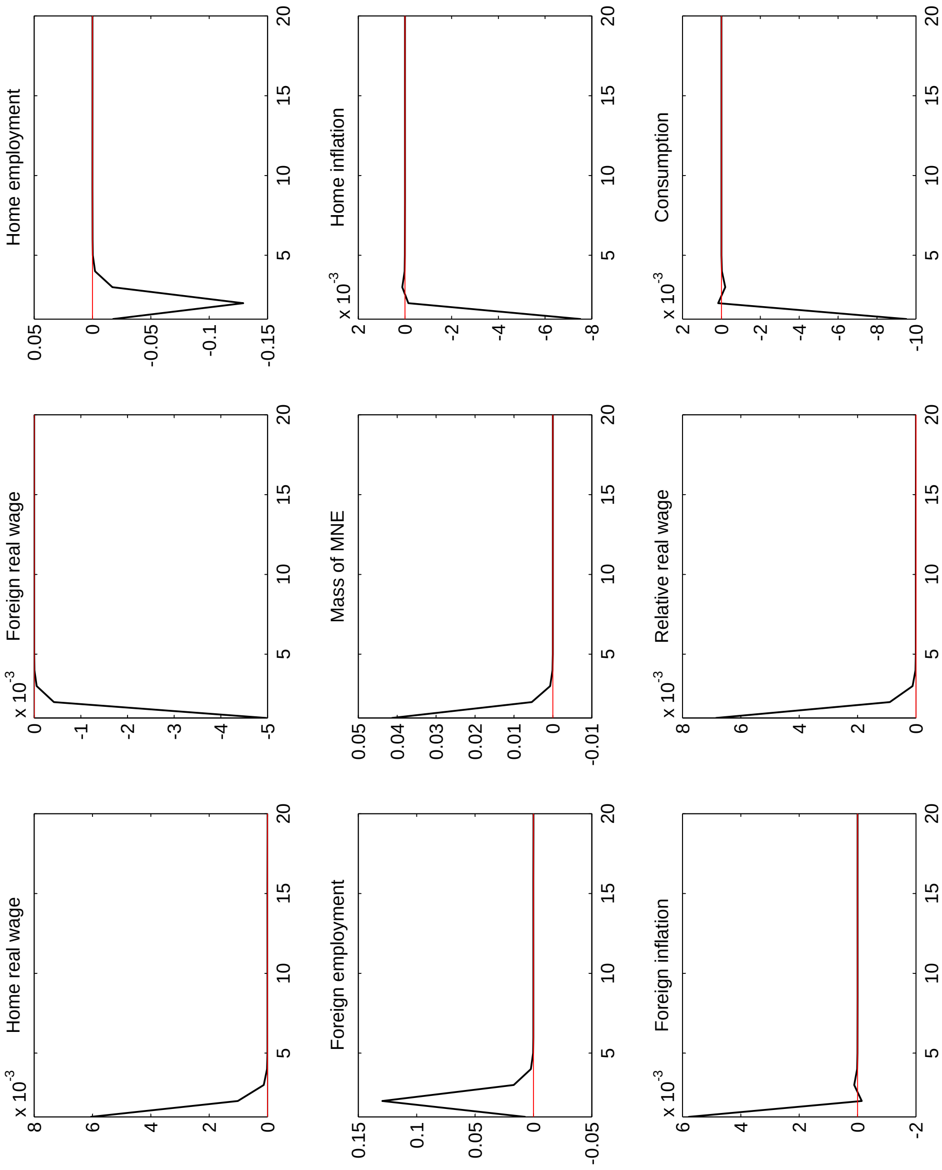


Figure 1: IRFs after a nominal shock: Orthogonalized shock to ν_t

8.2 IRFs after a real shock: Orthogonalized shock to ϵ_t

Next figure shows IRFs of variables of the system after an orthogonalized shock to the aggregate Home productivity. Upon impact, real wages in both countries increase, as the positive productivity shock increases as well productivity of those Home firms who are producing in the Foreign country as multinationals in the current period. As consumptions in both countries increase, consumption-leisure substitution dictates that workers now prefer to work less than before, although sticky wage prevents the necessary adjustment to happen in the very first period.

The lower two panels in the central column show that the old positive relationship between the changes in FDI flows and the real exchange rate is preserved under the aggregate productivity shock. When Home experiences a positive productivity shock, everything else equal, the marginal firm with the cutoff productivity $\tilde{\varphi}_t$ will earn positive profit by moving abroad. Moreover, as all home firms become more productive, the Home real wage goes up relative to the Foreign. The mass of multinationals increases under these effects, implying an increased FDI flow from Home to Foreign.

Thus we have established that the changes in FDI flows and exchange rate exhibit the old positive correlation following both types of shocks.

9 Welfare Effects of output gap response

This section is dedicated to the discussion of good monetary policy in the model environment. We re-examine the desirability of responding to output gap when cross-border relocation is allowed. As we show in section 2.1, output gap targeting when real shocks present induces a cost that resembles an enlarged volatility for nominal shocks. However, intuition suggests that when cross-border firm relocation is allowed, targeting output gap is similar to targeting the employment gap, which is the major culprit of welfare-loss under nominal shocks. Figure (2.3) presents a piece of evidence that supports this intuition. The blue line depicts the IRF of mass of MNEs in the baseline case, when $\rho_y = 0$, under a shock to the Home interest rate whose magnitude is one standard deviation. The green line, on the other hand, depicts the same IRF when the output response coefficient is set to a moderate 0.1. In the baseline simulation, more than 4% of Home firms become new multinationals following the shock, whereas setting ρ_y to 0.1 decreases the new multinationals to less than 1%.

The analysis above suggests that output gap targeting is an effective way to tame the short run fluctuations in FDI flows. The more important question is then, how is the expected welfare affected by output targeting? The answer to this question obviously depends on how large is the gain compared to the cost of mistakenly responding to the deviation of the natural level of output from steady state. Equation (38) points out that the cost is larger when that deviation is larger. Thus a reasonable guess is that the relative benefit of output gap targeting is increasing with the relative importance of nominal shocks compared with real shocks, as reducing the volatility of FDI flows caused by nominal shocks is welfare-improving, yet correcting for those natural level deviations caused by real shocks is counter-productive. Next we formally verify this conjecture by computing the expected world utility corresponding to different values of output response coefficient in a second order approximation.

The expected world utility is computed in the same way as in chapter one. We simulate the model in second order approximation for 2000 periods, drop the first 200 periods, and compute the average period utility. We repeat this process to make sure that we get stable results.

Figure (2.4), (2.5) and (2.6) depict how the expected period world utility changes as the output response coefficient ρ_y is increased from the baseline value of zero to one. The upper blue line represents the steady state level of world welfare and the lower green line represents the expected world utility at different values of

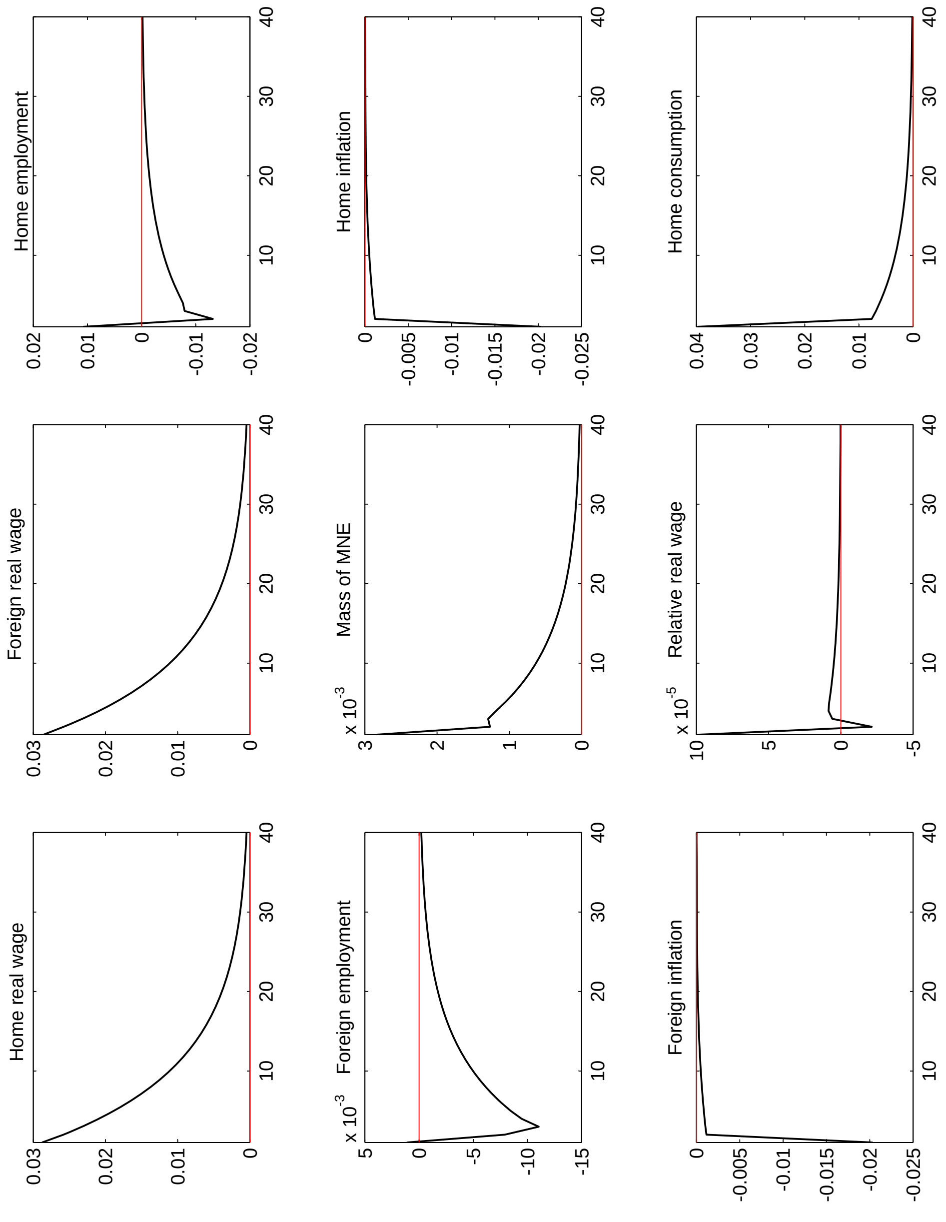


Figure 2: IRFs after a real shock: Orthogonalized shock to ϵ_t

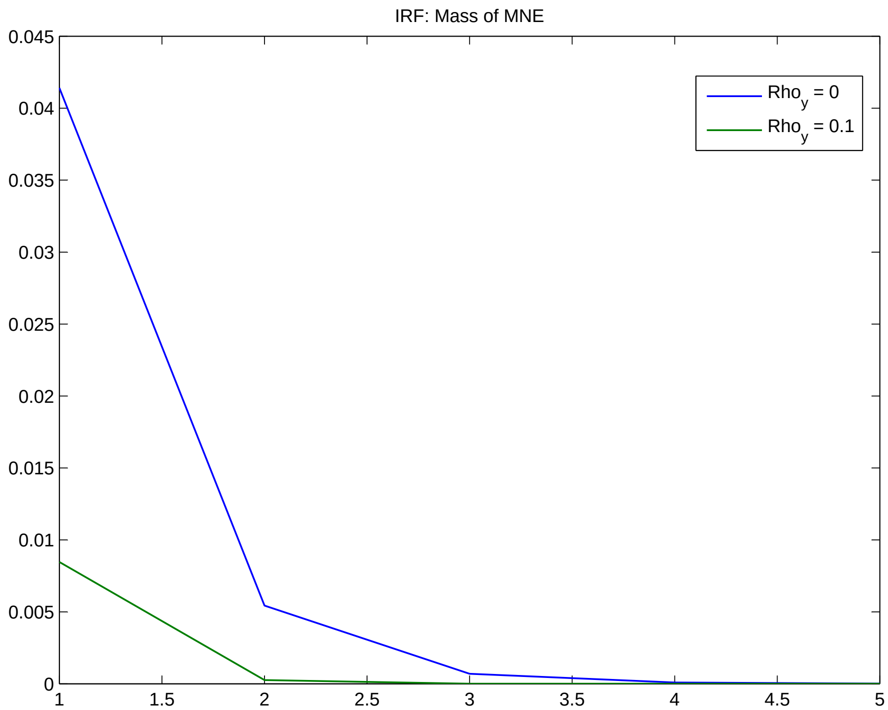


Figure 3: IRF of Mass of MNE in cases $\rho_y = 0$ and $\rho_y = 0.1$

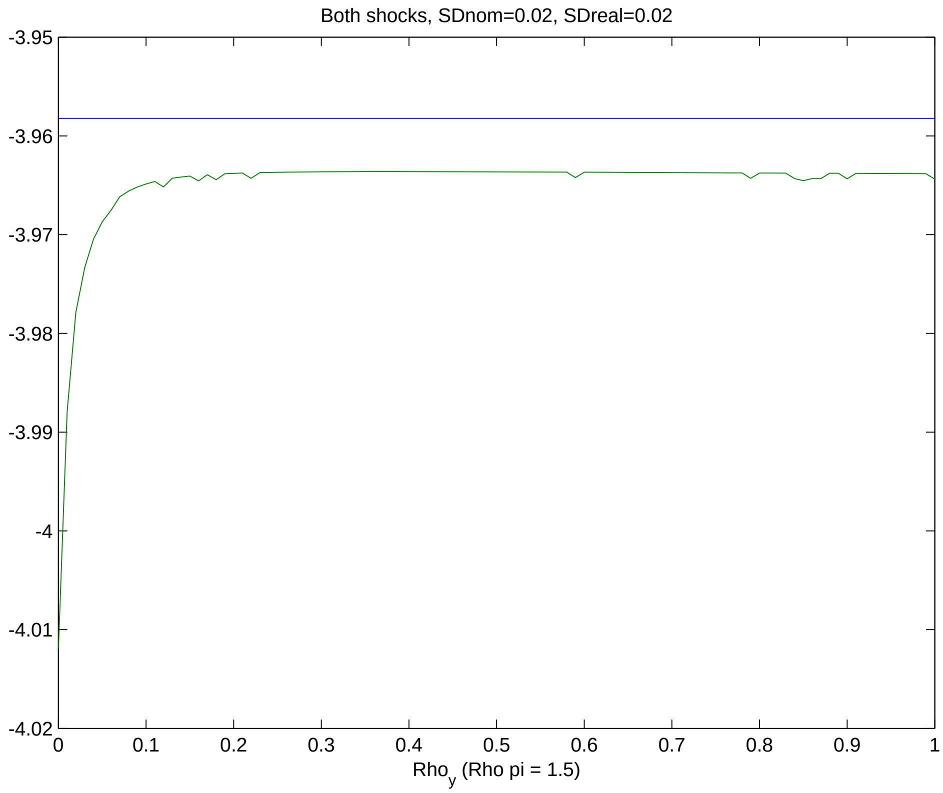


Figure 4: Expected world utility as a function of ρ_y

ρ_y. In the baseline case in figure (2.4), when both nominal and real shocks have the same standard deviation of 0.02, increasing the output response coefficient improves utility at first, and inconspicuously decreases it when we increase ρ_y further.

That pattern is more apparent when the importance of the productivity shocks are increased. To contrast with the last case, we set the standard deviation of the productivity shocks to 0.1, while keeping the volatility of nominal shocks the same as before. The world utility in this environment is depicted in figure (2.5). As ρ_y increases from zero, the expected world utility first increases, and then drops. This is because ρ_y serves as a multiplier to the deviations of the natural level of output in (38). When ρ_y is small, the cost of output gap response is negligible, but it does a beneficial job of discouraging the FDI fluctuations caused by nominal shocks. As we continue increasing the output response coefficient though, its benefit is exhausted and the importance of the cost takes over. Fixing the magnitude of the real shocks, increasing ρ_y directly contributes to the volatility of the system. The lesson from this case is then that, when real shocks are volatile compared with nominal shocks, there exists an optimal level of output response strength at which the expected world utility is maximized.

Figure (2.6) is a case when the nominal shocks are relatively more volatile than the real shocks. The distortion of output gap targeting appears to be substantially tamed, and increasing ρ_y pushes the expected utility very close to its steady state level.

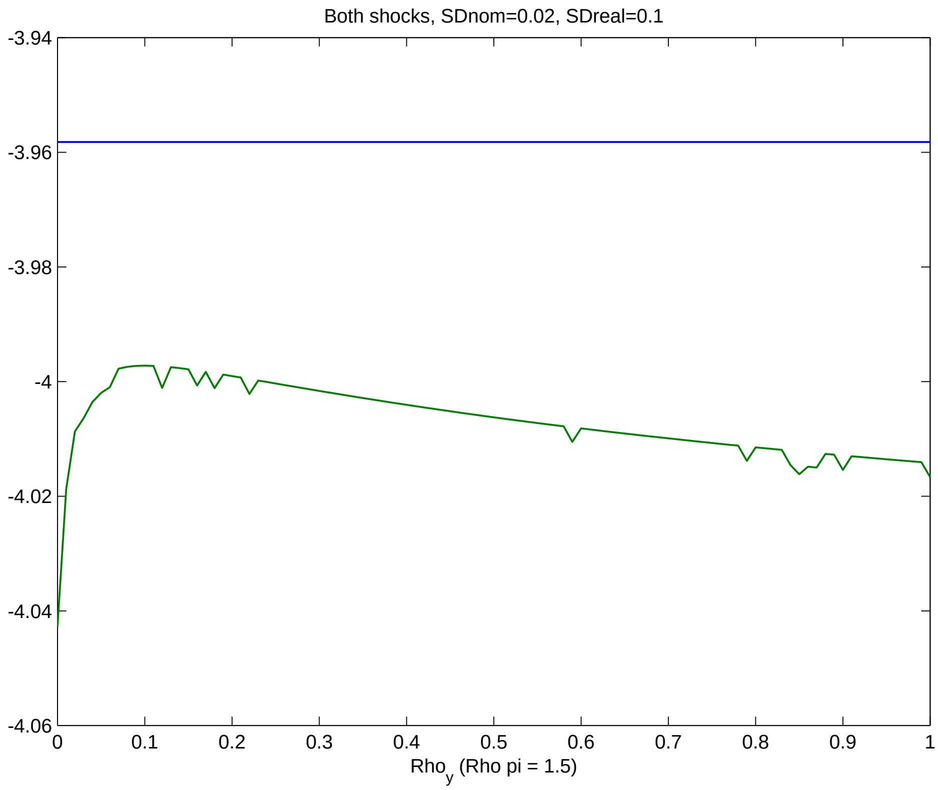


Figure 5: Expected world utility, volatile real shocks

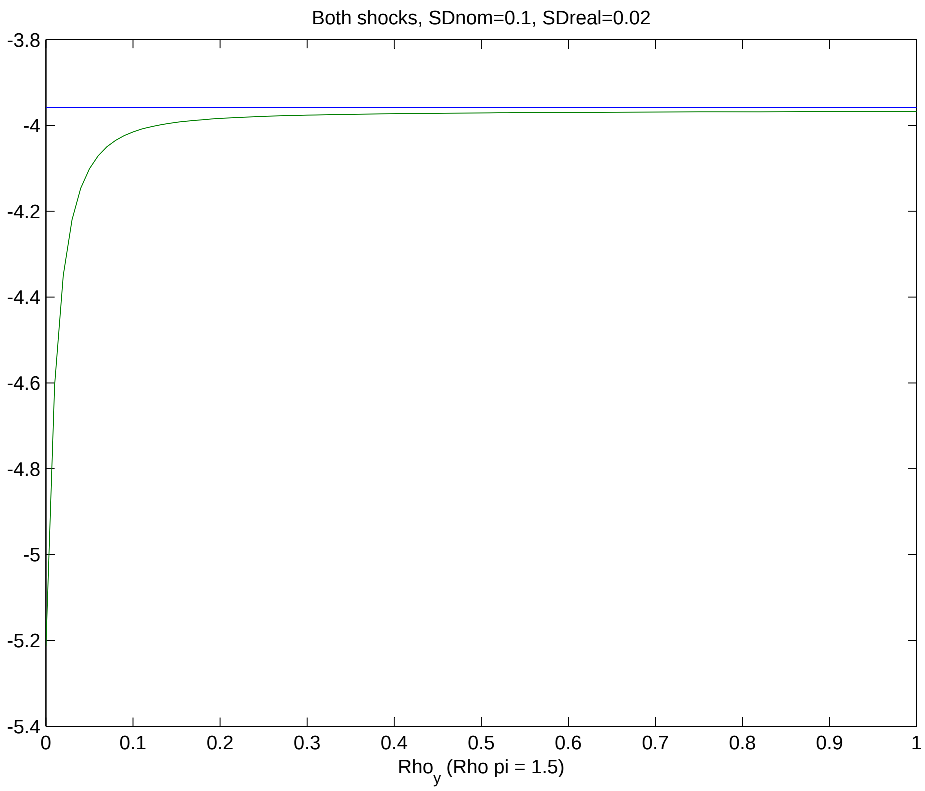


Figure 6: Expected world utility, volatile nominal shocks

To summarize, when we increase the output gap response coefficient, the expected world utility in second order approximation exhibit different patterns under different assumptions regarding the relative volatility of real and nominal shocks. Theory suggests that output response causes more distortion when the real shocks are volatile, as the natural level of output experiences more fluctuations. The simulation results in this section conform with this intuition. Although this is a simple analysis, one implication seems to be apparent: When multinational firms present, setting output response coefficient to zero is not optimal.

10 Conclusions

This paper explains the positive correlation between FDI inflow and exchange rate by studying firm's location decision-making in an open economy macro model. Monetary shocks are shown to affect the level of real wages when nominal friction exists in the wage-setting process. We argue that when firms' motivation of relocation come from a lower production cost abroad, the change in real wages caused by nominal frictions leads to change in the equilibrium production location distribution of firms, and thus generating the observed positive correlation of FDI inflow and the exchange rate.

By comparing the short run behaviors of the relative real wage and aggregate employments, we find that the option to relocate abroad has ambiguous welfare implications. While speeding up the convergence of the economy back to the efficient long run equilibrium, cross-border relocation exacerbates the inefficiencies in aggregate working hours. And the two countries are affected asymmetrically.

A further question to ask is how the volatility of exchange rate affects firms' location decisions. Empirical literature gives mixed answers to this question. The issue can be addressed by taking a second order approximation of the system. An analysis in second order also allows us to examine the welfare effects of FDI flows more carefully. Currently, we note the adverse effects of FDI flows on the aggregate employment, but it causes also a second inefficiency. MNE production brings aggregate real wages back to their long run equilibrium at a faster rate, but under the Calvo type nominal rigidities, this can only be done by enlarging the variance of the individual households' wage distribution, as the wage re-setters will have to ask for a higher wage rate for the aggregate wage to raise faster. Erceg, Henderson, and Levin (2000) have shown that the variance of the wage distribution negatively affects the expected utility, but capturing the effects of the second moment requires a second order approximation of the utility function.

We augment the baseline model with interest rate rules and aggregate productivity shocks to examine the optimality of output gap targeting in this chapter. Simulation indicates that the positive relationship between the changes in FDI flows and exchange rate is robust under aggregate productivity shocks. Furthermore, as each country's aggregate employment is directly connected to its aggregate output, adopting a monetary policy that takes into account the output gap is an effective way to tame the fluctuations of FDI flows over the business cycle. As a result, output gap targeting helps reducing the welfare loss associated with these cross-border firm relocation found in chapter one.

The net benefit of output gap response is increasing in the relative volatility of nominal shocks compared to the real ones. When real shocks are more volatile, there exists an optimal output response coefficient at which the expected world utility is maximized. Moreover, we have shown that the distortion caused by mistakenly responding to the deviation of the natural level of output is proportional to the magnitude of the response coefficient. Thus, when the coefficient is small, the loss is negligible, but its marginal benefit is largest when the coefficient is small. As a result, in all our simulations, it is always optimal to respond to output gap to some extent.

The baseline model features sticky nominal wage, but not sticky nominal price. This constitutes a major deviation from previous New Keynesian literature. It will be interesting to add nominal friction in the goods market. Also, as capital input is fixed at one in the baseline model, it is virtually assumed that capital can be moved across border in one period. This is true for some industries, e.g. phone service, but falls short in terms of reality in general. It is interesting to add a mechanism that induces sluggish capital movement.

For example, a convex cost of capital adjustment should slow down the pace of cross-border firm relocation, and make the model's dynamics more realistic.

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A Proof: FDI flows from the country with high average productivity to the country with low average productivity

We prove by contradiction.

Case 1: Assume there is no cross-border investment in the steady state.

Given that the productivities are distributed on $[0, \infty)$, the breakeven condition (12) implies that the real wages must equalize. Thus (22) becomes

$$\frac{\bar{C}}{\bar{C}^*} = \left(\frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}}\right)^{-\frac{\phi}{\sigma}} \tag{44}$$

where

$$\bar{\varphi} \equiv \left[\int_1^{\infty} \varphi^{\mu-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\mu-1}}$$

is an uncondition average productivity of Home firms. By assumption, $\bar{\varphi} > \bar{\varphi}^*$. Therefore (44) implies $\bar{C}/\bar{C}^* < 1$. On the other hand, without foreign production, each country gets their domestic production. Setting $\bar{\varphi}_H^{*\mu-1} = 0, \overline{\varphi}_H = \bar{\varphi}$ and $G(\overline{\tilde{\varphi}}) = 1$ in the two budget constraint (23) and (24) and then divide, we get

$$\frac{\bar{C}}{\bar{C}^*} = \frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}} > 1,$$

which leads to a contradiction with the prediction of labor market.

Case 2: Assume that in steady state, FDI flows from Foreign to Home.

If this is true, the breakeven condition (12) implies that the real wage must be higher in Foreign, i.e. $\bar{\omega}^* > \bar{\omega}$. We have the counterpart of (22)

$$\left(\frac{\bar{C}}{\bar{C}^*}\right)^{\frac{\sigma}{\phi}} = \left(\frac{\bar{\omega}}{\bar{\omega}^*}\right)^{\frac{1}{\phi}+\mu} \left(\frac{\overline{\varphi}_F^{*\mu-1}}{\overline{\varphi}_F^{\mu-1} + \bar{\varphi}^{\mu-1}}\right), \tag{45}$$

where

$$\overline{\varphi}_F^* \equiv \left[\int_{\varphi_m^*}^{\overline{\tilde{\varphi}}^*} \varphi^{\mu-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\mu-1}},$$

and

$$\overline{\overline{\varphi}}_F^* \equiv \left[\int_{\overline{\tilde{\varphi}}^*}^{\infty} \varphi^{\mu-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\mu-1}}$$

are the aggregate productivity of Foreign firms who produce domestically and Foreign firms who produces abroad, respectively, and $\overline{\tilde{\varphi}}^*$ is the cutoff productivity. Since $\bar{\varphi}^{\mu-1} > \bar{\varphi}^{*\mu-1} > \overline{\varphi}_F^{*\mu-1}$, and $\bar{\omega}^* > \bar{\omega}$, (45) implies that $\bar{C}/\bar{C}^* < 1$.

By symmetry, the two budget constraints (23) and (24) become

$$\bar{C} = \left(\frac{\mu}{\mu-1}\right)^{-\mu} \bar{\omega}^{1-\mu} \bar{C}^w \overline{\varphi}_F^{\mu-1} + \left(\frac{\mu}{\mu-1}\right)^{1-\mu} \bar{\omega}^{1-\mu} \bar{C}^w \bar{\varphi}^{\mu-1},$$

$$\bar{C}^* = \left(\frac{\mu}{\mu-1}\right)^{1-\mu} \bar{\omega}^{*1-\mu} \bar{C}^w \overline{\varphi}_F^{*\mu-1} + \left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu} \bar{\omega}^{1-\mu} \bar{C}^w \overline{\varphi}_F^{\mu-1} - \left(1 - G^*\left(\overline{\tilde{\varphi}}^*\right)\right) F.$$

The labor market equilibrium implies $\bar{C}/\bar{C}^* < 1$, and we must have

$$\begin{aligned} &\left(\frac{\mu}{\mu-1}\right)^{-\mu} \bar{\omega}^{1-\mu} \bar{C}^w \overline{\varphi}_F^{\mu-1} + \left(\frac{\mu}{\mu-1}\right)^{1-\mu} \bar{\omega}^{1-\mu} \bar{C}^w \bar{\varphi}^{\mu-1} \\ < \; &\left(\frac{\mu}{\mu-1}\right)^{1-\mu} \bar{\omega}^{*1-\mu} \bar{C}^w \overline{\varphi}_F^{*\mu-1} + \left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu} \bar{\omega}^{1-\mu} \bar{C}^w \overline{\varphi}_F^{\mu-1} - \left(1 - G^*\left(\overline{\tilde{\varphi}}^*\right)\right) F \end{aligned}$$

Then the relation below must be true:

$$\left(\frac{\mu}{\mu-1}\right)^{-\mu} \overline{\varphi}_F^{\mu-1} + \left(\frac{\mu}{\mu-1}\right)^{1-\mu} \bar{\varphi}^{\mu-1} < \left(\frac{\mu}{\mu-1}\right)^{1-\mu} \frac{\bar{\omega}^{*1-\mu}}{\bar{\omega}^{1-\mu}} \overline{\varphi}_F^{*\mu-1} + \left(\frac{1}{\mu-1}\right)\left(\frac{\mu}{\mu-1}\right)^{-\mu} \overline{\varphi}_F^{\mu-1}$$

Rearranging to get

$$\mu \bar{\varphi}^{\mu-1} < \mu \frac{\bar{\omega}^{*1-\mu}}{\bar{\omega}^{1-\mu}} \overline{\varphi}_F^{*\mu-1} + \overline{\varphi}_F^{\mu-1} - (\mu-1)\, \overline{\varphi}_F^{\mu-1} \tag{46}$$

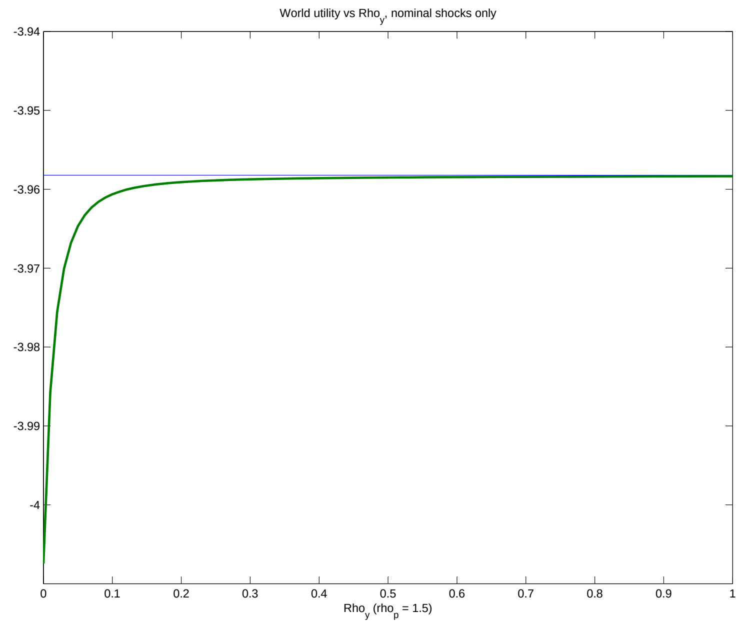

Figure 7: Expected world utility as a function of ρ_y

as $\bar{\omega}^* > \bar{\omega}$ and $\mu > 1$, the RHS of the equation satisfies

$$\begin{aligned} \mu \frac{\bar{\omega}^{*1-\mu}}{\bar{\omega}^{1-\mu}} \overline{\overline{\varphi}}_F^{*\mu-1} + \overline{\overline{\varphi}}_F^{\mu-1} - (\mu-1)\,\overline{\overline{\varphi}}_F^{\mu-1} &< \mu \overline{\overline{\varphi}}_F^{*\mu-1} + \overline{\overline{\varphi}}_F^{\mu-1} - (\mu-1)\,\overline{\overline{\varphi}}_F^{\mu-1} \\ &< \mu \left(\overline{\overline{\varphi}}_F^{*\mu-1} + \overline{\overline{\varphi}}_F^{\mu-1} \right) - (\mu-1)\,\overline{\overline{\varphi}}_F^{\mu-1} \end{aligned}$$

But note that

$$\overline{\overline{\varphi}}_F^{*\mu-1} + \overline{\overline{\varphi}}_F^{\mu-1} = \bar{\varphi}^{*\mu-1} < \bar{\varphi}^{\mu-1}$$

Thus (46) can't be true. Therefore Case#2 can't be true. The only possible equilibrium is for the country with high productivity to relocate to the country with low productivity.

B World utility in an economy with only nominal shocks

With only nominal shocks of the kind specified in (35), increasing strength of output gap response uniformly increasing the expected world utility. This is depicted in figure (B.1). The reason underlying the result is intuitive: Output gap response automatically takes into account the changes in aggregate employments of both countries from their steady state levels. For example, when Home firms are attracted by a lower wage rate abroad to become short run multinationals in the Foreign country, the relocation increases demand for Foreign labor, and at the same time decreases the demand for Home labor. These changes in aggregate employment are the sources of the inefficiency outlined in chapter one. When output gap response is enforced, these aggregate employments deviations will trigger an interest response, as GDP is directly linked to the aggregate employment. As a result, output gap response becomes an effective tool in restricting short run cross-border relocation. Note that this is a special case of the more general model with both shocks in the text, when the volatility of the real shocks are set to zero.