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Abstract

This article revisits the Lucas Illustration of the Quantity Theory of Money (QTM) and investigates whether it holds in the US monetary market. The findings confirm that QTM does not hold in the short run and the Cash-in-Advance (CIA) model fails to replicate these empirical results because the economy under the CIA framework reacts too quickly to monetary shocks. To correct for this failure, this article incorporates the financial intermediary and default as an equilibrium phenomenon into the original CIA model. The results suggest that the modified CIA model fits the short-run QTM abnormality better compared to the original model.

Keywords: Cash-in-Advance, default, financial intermediary, Quantity Theory of Money

JEL Classification: E41, E51

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I. Introduction

After the Quantity Theory of Money (QTM) was first formulated (David Hume, 1742), many empirical studies investigated whether this theory holds in the real world. Past empirical studies (Lucas, 1980; Bullard, 1994; Diaz-Gimenez and Kirkby, 2014) observed that QTM holds in the long run but does not hold in the short run in the US monetary market. In addition, three standard monetary models such as the Cash-in-Advance (CIA), the New Keynesian, and the Search-Money models failed to replicate these results (i.e., gave tighter predictions of QTM) in the short run because prices respond too quickly to changes in the money growth rates (Hodrick *et al.*, 1991; Diaz-Gimenez and Kirkby, 2014).

The main reason for the replication failure under the original CIA model is that every change in the growth rate of money is both universal and simultaneous by construction. Under the representative agent setup, when money enters the economy at one point and reaches the representative agent, it has no space to be spread around. As a result, it is very hard to achieve a sluggish response in prices. To generate the sluggish price response, past studies have attempted to embody liquidity effects (Christiano, 1991; Christiano and Eichenbaum, 1995), or construct a model for inventory of money on account of multiple-period CIA constraints (Alvarez *et al.*, 2009).

As another means of improvement of the CIA model against QTM in the short run, we incorporate (i) agent heterogeneity, i.e., a financial intermediary, and (ii) default as an equilibrium phenomenon into the original model to slow down the quick price response to monetary shocks. The results suggest that QTM can be explained better with this modified CIA model. This better replication result comes from two factors. First, incorporating the financial intermediary segments the financial market and prevents households from completely hedging the risk against monetary shocks. In addition, these shocks are exaggerated by a financial intermediary through a financial accelerator effect (Bernanke *et al.*, 1999) and pro-cyclical lending behaviour (Gourio, 2012; Zhang, 2005; Storesletten, 2007). Thus, the quick price response under the original CIA slows down.

Second, financial friction defined as endogenous default amplifies monetary shocks as it distorts the investments in response to these shocks. Specifically, loan losses diminish credit supply, which suppresses prices and income further. The suppression in prices and income, in turn, makes default even worse (Tsomocos, 2003; Dubey *et al.*, 2000; Geanakoplos, 1997). As a result, endogenous default makes monetary shocks amplified and transmitted into the economy and distorts investment much bigger. This process helps to loosen the tight prediction of QTM. In sum, the two modeling ingredients are the driving forces of better fit to the results (i.e., the slower reaction of the economy to monetary shocks as observed in empirical studies).

II. QTM and Data

Based on QTM, the Lucas Illustration suggests that plotting the money growth rate (x-axis) against the sum of the inflation rate and output growth rate (y-axis) must be on the 45 degree line.[1] To identify what QTM looks like in the US monetary market, we extract the quarterly data, annual growth rates of M1, M2, Consumer Price Index (CPI), and Gross National Product (GNP) from Federal Reserve Economic Data (FRED) from 1971:Q1 to 2014:Q2. Then, we plot the data on the defined x-y plane in the short run. Since it also needs to quantify the Lucas Illustration, we first compute the slope of an ordinary least square (OLS) from the plotted data (Whiteman, 1984). The OLS coefficient will be close to one if QTM holds and non-unity values otherwise.

In addition, we compute the average Cartesian distance of the plotted points from the 45 degree line that goes through the grand mean of the unfiltered data (Diaz-Gimenez and Kirkby, 2014). It is calculated as the following:

$$D45 = \frac{1}{\sqrt{2T}} \sum_{i=1}^{T} |x_i - y_i - (\bar{x} - \bar{y})| \tag{1}$$

where x_i is the corresponding observation of the money growth rate, and y_i is the value

[1] We include the growth rate of output to the original Lucas formula because the inflation rate is moderate but output growth is relatively high in our sample period.

of the i^{th} observation of the sum of the inflation rate and output growth rate. Also, $\bar{x}$ and $\bar{y}$ are the average values of the unfiltered x_i and y_i. The average distance will be close to zero if QTM holds and nonzero otherwise.

Lastly, to figure out the QTM pattern in the long run, we use a theoretical filter following the Lucas procedure (Lucas, 1980). We identify the long run with the low frequency fluctuations of the inflation rate, output growth rate, and money growth rate. To remove the high frequency fluctuations, we transform the original time series data using a two-sided and exponentially-weighted moving average filter (Sargent and Surico, 2011). The transformation equation is as follows:

$$x_t(\beta) = \alpha \sum_{k=1}^{T} \beta^{|t-k|} x_k \tag{2}$$

where $\alpha = \frac{(1-\beta)^2}{1-\beta^2-2\beta^{(T+1)/2}(1-\beta)}$ is defined for $0 \leq \beta < 1$ and T is the number of observations in the time series data. After extracting the low frequency movements from the original time series, we plot the filtered series on the $x - y$ plane. In addition, we do the same exercises of calculating the OLS coefficients and $D45$ in the long run as in the short run.

III. Results

Panels A and B of Figure 1 illustrate QTM in the long run (from 1971:Q1 to 2014:Q2) with M1 and M2 as the monetary aggregate, when $\beta = 0.95$ (Lucas, 1980; Bullard, 1994). All four time series data (M1, M2, CPI, and GNP) are filtered as described above. This shows the clear pattern of the 45 degree line that QTM predicts, and the linear pattern is more evident with M2. The first two rows of Table 1 present two statistics quantifying these graphical patterns. The average distances from the 45 degree line that passes through the grand mean of the filtered data are close to zero (0.2276 for M1 and 0.1210 for M2) and smaller with M2. Also, the OLS coefficients are close to 1 (1.2987 for M1 and 1.2039 for M2) and the tighter prediction is made with M2. The results are particularly striking if the measure of money is broad, M2. The narrow monetary measure, M1, tends not to provide as convincing an illustration (Bullard, 1994).

<Figure 1 about here>

However, we cannot find the clear pattern of a 45 degree line that QTM predicts from the scattered plots in the short run. Panels C and D of Figure 1 represent a vague pattern of scattered plots from both M1 and M2 in the short run. The average distance from the 45 degree line in the bottom two rows of Table 1 is 3.5396 with M1 and 2.0517 with M2. These statistics are much greater compared to the estimates (close to zero) in the long run. Also, the OLS coefficients in the bottom two rows of Table 1 show that the slopes considerably deviate from one, which implies that QTM does not hold in the short run. All these results are consistent with past studies (Lucas, 1980; Bullard, 1994; Diaz-Gimenez and Kirkby, 2014).

<Table 1 about here>

Accordingly, we explore the extent to which QTM holds in the CIA framework. In this article, we use Cooley and Hansen (1989) as our baseline CIA framework because this model economy combines a CIA constraint and the standard neoclassical model of business cycles. The results from CIA provide the stronger QTM relationship compared to the US monetary data, especially in the short run. The first panel of Table 2 presents the two statistics of the equilibrium process of the CIA model and they support the Lucas Illustration. The average distance from the 45 degree line in the short run is 1.1809, which is much smaller in the CIA framework compared to the US monetary market (3.5396 for M1 and 2.0517 for M2). The slope is also closer to one (1.2467), which implies that QTM in the CIA framework displays much stronger prediction than the US monetary data. The reason the CIA model fails to replicate the QTM relationship is that the model economy reacts too quickly to monetary shocks relative to the data. In other words, it does not display enough short-run sluggishness in the response of prices.

To overcome the limitation that the original CIA model has, we incorporate a financial intermediary and endogenous default into the original model and have a better fit.

The results from these modified CIA models are presented in panels (2), (3), and (4) of Table 2. First, we introduce the risk-neutral financial intermediary and endogenous default (the second panel of Table 2) into the original CIA model, and then change the risk-neutral financial intermediary to a risk-averse one (the third panel of Table 2). Lastly, we increase the degree of sensitivity for a non-pecuniary default penalty in relation to the risk-averse financial intermediary (the fourth panel of Table 2). The results show much more similar patterns to the QTM relationship from the US data. Also, the two statistics change in such a way that the average distance of 45 degree line has increased, and the OLS coefficient has increased and deviated from unity. The fit improves as we incorporate the risk-averseness into the financial intermediary and increase the sensitivity of the non-pecuniary default penalty.[2]

<Table 2 about here>

These better estimates result from the sluggish price responses created by the risk-averse financial intermediary and financial frictions. First, with the existence of agent heterogeneity, it takes considerable time for the money to be spread around the economy after it is introduced into the model economy. Since money growth is now neither universal nor simultaneous, the price growth rate does not respond immediately to money growth rate change anymore. Thus, agent heterogeneity, especially financial intermediary, with market segmentation turns out to be a necessary condition for the model economy to display the price sluggishness.

In addition, a financial system incorporating an endogenous default amplifies distorted investments in response to monetary shocks and helps to display a less tight prediction of QTM in the short run. Specifically, a drop in credit supply due to loan losses suppresses prices and income further, which in turn makes default even worse. As a result, default makes shocks amplified and transmitted into the economy. The bigger distortion in investments due to endogenous default produces a weaker prediction of QTM.

[2]The functional form and implied parameter values are explained in detail in appendix A.

IV. Conclusion

This article revisits QTM in the US monetary market using a rich data set (quarterly data from 1971:Q1 to 2014:Q2). It confirms that QTM does not hold in the short run and that the existing CIA model has limitations in its ability to replicate that. This article contributes to the literature in the way that it incorporates agent heterogeneity (financial intermediary) and financial friction (endogenous default) into the original CIA model, and better explains the current QTM relationship from the data compared to the original CIA model. Future research modeling secured loans and considering heterogeneity across banks is recommended, as these are areas not considered in this study but that would generate a better model fit to QTM in the short run.

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Appendices

Appendix A. Cash-in-Advance Economies

A.1 The Model by Schorfheide (2000)

The model economy consists of a representative household, a firm, and a financial intermediary, which is a bank in this case. Output is produced according to a Cobb-Douglas production function,

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha} \tag{3}$$

where K_t denotes the capital stock (predetermined at the beginning of period t), N_t is the labor input, A_t is the Total Factor Productivity (TFP), α is output elasticity of capital, and $1-\alpha$ is output elasticity of labor.

The model economy is perturbed by two exogenous processes. Firstly, technology follows a stationary AR(1) process,

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + (1-\rho_A)\ln \bar{A} + \sigma_A \epsilon_{A,t} \\ \epsilon_{A,t} &\sim i.i.d.\ N(0,1) \end{aligned} \tag{4}$$

ρ_A refers to the AR(1) coefficient of technology and $\bar{A}$ indicates the steady state of technology. The innovation $\epsilon_{A,t}$ follows a $N(0,1)$, where σ_A denotes the standard deviation of innovations to $\ln A_t$.

The central bank lets the money stock M_t grow at rate $m_t = M_{t+1}/M_t$. m_t is a shifter to intertemporal money supply growth that follows an AR(1) process:

$$\begin{aligned} \ln m_t &= \rho_m \ln m_{t-1} + (1-\rho_m)\ln \bar{m} + \sigma_m \epsilon_{m,t} \\ \epsilon_{m,t} &\sim i.i.d.\ N(0,1) \end{aligned} \tag{5}$$

Equation (5) can be interpreted as a simple monetary policy rule without feedback. $\epsilon_{m,t}$ is the monetary policy shock. To put it in more detail, $\epsilon_{m,t}$ to the monetary policy follows a $N(0,1)$ that captures unexpected changes of the money growth rate due to normal policy making (Sims *et al.*, 1982), and σ_m measures the standard deviation of innovations.

At the beginning of period t, the representative household inherits the entire money stock of the economy, M_t. The aggregate price level is denoted by P_t. In the standard CIA model, all decisions are made after, and therefore completely reflect, the current period's surprise change in money growth and technology. The household determines how much money D_t to deposit in the bank, while these deposits earn interest at the rate $R_{H,t}$. The bank receives household deposits and a monetary injection X_t from the central bank, which it lends to the firm at rate $R_{F,t}$.

The firm starts to produce and hires labor services from the household. After the firm produces its output, it uses the money borrowed from the bank to pay wages $W_t H_t$, where W_t is the nominal hourly wage and H_t is hours worked. The household's cash balance increases to $M_t - D_t + W_t H_t$. The CIA constraint implies that a consumer must hold some cash in advance to buy goods. The firm's net cash inflow is paid as dividend F_t to the household. Moreover, the household receives back its bank deposits inclusive of interest and the net cash inflow of the bank as dividend B_t.

In period t, the household chooses consumption C_t, hours worked H_t, and non-negative deposits D_t to maximize the expected sum of discounted future utility. Resultantly, it solves the problem:

$$\max_{\{C_t, H_t, M_{t+1}, D_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{(1-\phi)\ln C_t + \phi \ln(1-H_t)\right\}$$

$$s.t.$$

$$P_t C_t \leq M_t - D_t + W_t H_t \tag{6}$$

$$0 \leq D_t \tag{7}$$

$$M_{t+1} = (M_t - D_t + W_t H_t - P_t C_t) + R_{H,t} D_t + F_t + B_t \tag{8}$$

where β refers to the discount factor, ϕ measures the marginal rate of substitution between leisure and consumption, and $E_0(\cdot)$ is the expectation operator conditional on date 0.

The first constraint spells out the CIA constraint including wage revenues, the second the inability to borrow from the bank, and the third the intertemporal budget constraint emphasizing that households accumulate the money from total inflows made up of the money they receive from firms F_t and from banks B_t.

The firm chooses the next period's capital stock K_{t+1}, labor demand N_t, dividends F_t and loans L_t. Since households value a unit of nominal dividends in terms of the consumption it enables during period $t+1$, and firms and the financial intermediary are owned by households, date t nominal dividends are discounted by date $t+1$ marginal utility of consumption. Thus, the firm solves the problem:

$$\max_{\{F_t, K_{t+1}, N_t, L_t\}} \qquad E_0 \sum_{t=0}^{\infty} \beta^{t+1} \frac{F_t}{C_{t+1} P_{t+1}}$$

$$s.t.$$

$$W_t N_t \leq L_t \tag{9}$$

$$F_t = L_t + P_t \left[K_t^{\alpha} \left(A_t N_t \right)^{1-\alpha} - K_{t+1} + (1-\delta) K_t \right] - W_t N_t - R_{F,t} L_t \tag{10}$$

The first constraint the firm faces reflects the fact that the firm finances its current period wage bill $W_t N_t$ by borrowing L_t. The second constraint says that the firm balances paying the household larger dividends and accumulating more capital. Thus this constraint links this decision with labor demand and loan demand using the Constant Return to Scale (CRS) production function $Y_t = K_t^{\alpha} \left(A_t N_t \right)^{1-\alpha}$ for $(0 < \alpha < 1)$, and the law of motion of capital defines gross investment $I_t = K_{t+1} - (1-\delta) K_t$ for $(0 < \delta < 1)$, as well as goods market equilibrium $C_t + I_t = Y_t$.

The financial intermediary solves the trivial problem. The bank maximizes the expected infinite horizon discounted stream of dividends it pays to households:

$$\max_{\{B_t, L_t, D_t\}} \qquad E_0 \sum_{t=1}^{\infty} \beta^{t+1} \frac{B_t}{C_{t+1} P_{t+1}}$$

$$s.t.$$

$$L_t \leq X_t + D_t \tag{11}$$

$$B_t = D_t + R_{F,t} L_t - R_{H,t} D_t - L_t + X_t \tag{12}$$

where $X_t = M_{t+1} - M_t$ is the monetary injection. Banks receive cash deposits D_t from households and a cash injection X_t, and then use these funds to disburse loans to the firms L_t, on which they make a net return of $R_{F,t}$. The second constraint simply defines the cash flow balances of the bank.

A.2 The Model by Ahn *et al.* (2014)

The baseline model, however, has neglected the existence of default, a significant issue in the recent crisis. The possibility of default on any debt obligations underscores the necessity of CIA constraints. The interaction of liquidity and default justifies fiat money as the stipulated means of the exchange. Otherwise, the mere presence of banks without possibility of default or any other financial friction in equilibrium may become a veil without affecting real trade and final equilibrium allocation. Thus, we introduce endogenous default via CIA constraints based on the baseline model to properly capture the fundamental aspect of liquidity and how it interacts with default to affect the real economy.

Following Shubik and Wilson (1977) and Dubey *et al.* (2005), we modeled the default that arises as an equilibrium phenomenon, because agents are allowed to choose what fraction to pay from their outstanding debt. The cost of default is modeled by a non-pecuniary penalty that reduces utility, instead of directly reducing an individual's ability to borrow after the debtor defaults on a loan obligation.

According to the discussion, the amount that the bank has to repay on its liability has to be adjusted for the bank's repayment rate $\nu_{B,t}$. In this sense, instead of receiving full amount $R_{H,t}D_t$, household receives $\nu_{B,t}R_{H,t}D_t$. Thus, in the model with endogenous default, equation (8) becomes

$$M_{t+1} = (M_t - D_t + W_t H_t - P_t C_t) + \nu_{B,t} R_{H,t} D_t + F_t + B_t \tag{13}$$

The firms, as we mentioned in A.1, are debtors of loans from banks. We introduce a variable $\nu_{F,t}$, the proportion firms actually pay back. Thus, the actual cash flow concerning the interest paid to the banks becomes $\nu_{F,t}L_t R_{F,t}$ instead of $L_t R_{F,t}$. Additionally, since firms are allowed to default which will be injurious to their reputation, their utility function will be reduced to some extent and the non-pecuniary default penalty describes this reputation cost in the firm's utility function

$$\frac{c_F}{1+\eta_F}\left[\left(1-\nu_{F,t}\right)R_{F,t}\frac{L_t}{M_t}\right]^{1+\eta_F}$$

where c_F and η_F denote the coefficient and the sensitivity of non-pecuniary default penalty for firms.

So in the extended model, the utility function of firms in the baseline model changes to

$$\max_{\{F_t,K_{t+1},N_t,L_t,\nu_{B,t}\}} \qquad E_0\sum_{t=1}^{\infty}\beta^{t+1}\left\{\frac{F_t}{C_{t+1}P_{t+1}}-\frac{c_F}{1+\eta_F}\left[\left(1-\nu_{F,t}\right)R_{F,t}\frac{L_t}{M_t}\right]^{1+\eta_F}\right\}$$

Accordingly, the budget constraint of firms (10) changes to

$$F_t = L_t + P_t\left[K_t^{\alpha}\left(A_tN_t\right)^{1-\alpha} - K_{t+1} + \left(1-\delta\right)K_t\right] - W_tN_t - \nu_{F,t}L_tR_{F,t} \qquad (14)$$

We assume that banks are risk-averse instead of risk-neutral in the model with endogenous default. Since, in this framework, banks work as a financial intermediary and use deposit flow from households to provide loans to the firm, they are faced with monetary uncertainty. Besides, they also bear the default risk on loans to the firm. The assumption of risk-averse banks is in consistent with risk-averse behavior of banks (Niehans and Hewson, 1976; Niehans, 1978). Furthermore, Ratti (1980) finds supporting evidence for the hypothesis of risk-averse banks by presenting an analysis of a quasi-risk-averse bank facing uncertainty with respect to demand deposit flows and default risk on loans.

The changes of banks' utility function are similar to those of the firms but generalized. As we discussed above, $\nu_{B,t}$ is the repayment rate of banks and

$$\frac{c_B}{1+\eta_B}\left[\left(1-\nu_{B,t}\right)R_{H,t}\frac{D_t}{M_t}\right]^{1+\eta_B}$$

, as we defined in the firm sector, measures the impairment of banks' reputation cost due to the default. c_B and η_B represent the coefficient and the sensitivity of non-pecuniary default penalty for banks.

Thus, the utility function of the banks changes to

$$\max_{\{B_t, L_t, D_t, \nu_{B,t}\}} \quad E_0 \sum_{t=1}^{\infty} \beta^{t+1} \left\{ \frac{1}{1-\gamma} \left(\frac{B_t}{C_{t+1} P_{t+1}} \right)^{1-\gamma} - \frac{c_B}{1+\eta_B} \left[(1-\nu_{B,t}) R_{H,t} \frac{D_t}{M_t} \right]^{1+\eta_B} \right\}$$

where γ measures the degree of relative risk aversion that is implicit in the utility function.

The banks only pay $\nu_{B,t} R_{H,t} D_t$ to the households, while banks only receive $\nu_{F,t} R_{F,t} L_t$ from the firm. As a result, the budget constraint of banks changes to

$$B_t = D_t + \nu_{F,t} R_{F,t} L_t - \nu_{B,t} R_{H,t} D_t - L_t + X_t \tag{15}$$

A.3 Calibration

We choose the discount factor $\beta = 0.99$ as a default choice. For the depreciation rate, $\delta = 0.025$ is set to induce the appropriate capital-output ratio (Fernández-Villaverde *et al.*, 2010). We use $\alpha = 0.32$ for the capital share in the US production function (Schmitt-Grohé and Uribe, 2003) and set the AR(1) coefficient of technology $\rho_A = 0.95$ with the standard deviation $\sigma_A = 0.007$ following Cooley and Prescott (1995). Also, we normalize the value of total factor productivity in the steady state $\bar{A} = 1$. The AR(1) coefficient of the growth rate of monetary injections, $\rho_m = 0.6534$, as well as the standard deviation, $\sigma_m = 0.0098$, are estimated. Nason and Cogley (1994) estimates a marginal rate of substitution of $\phi = 0.773$ and we use this estimate for our calibration. As we can see, all the above calibrated values are well within the range of values used in the macroeconomic literature. As for c_F and c_B, we apply reverse engineering method and calibrate 267.75 and 448.95, respectively.[3]

Any differences in the outcome with respect to QTM can result from the different ways in which these four frameworks construct the money because all other possible sources of variations have been removed. Furthermore, we have used the identical solution method for the identical functional forms, parameter values, and exogenous processes.

[3] Reference Ahn *et al.*, 2014.

For our model economy simulations, we have used identical seeds for the random number generator. As a result, the sequences of the realizations of the random shocks are identical in all four model economies. In addition, we have removed the first 544 periods of each equilibrium realization in order that the equilibrium results are not influenced by the initial conditions. Then, we have drawn a sample of 174 quarterly observations to replicate the number of observations in our United States time series.

Appendix B. Figures and Tables

B.1 Figures

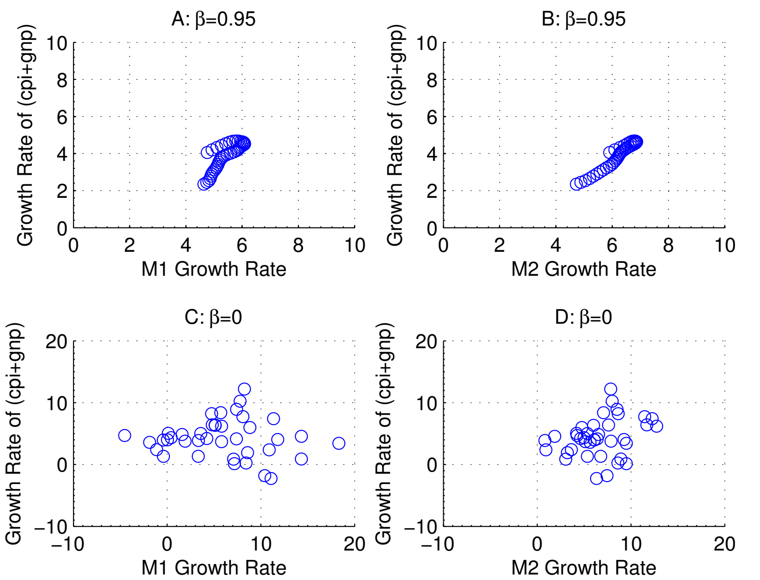

Fig. 1. The Quantity Theory of Money in US monetary markets

B.2 Tables

Table 1. The Quantity Theory of Money statistics in US monetary market

		Distance from 45 Degree Line	OLS Regression Coefficient
Long Run	M1	0.2276	1.2987
($\beta = 0.95$)	M2	0.1210	1.2039
Short Run	M1	3.5396	-0.0445
($\beta = 0$)	M2	2.0517	0.3018

Table 2. The Quantity Theory of Money statistics in CIA economies

		Distance from 45 Degree Line	OLS Regression Coefficient
(1)	Long Run	0.0469	0.9741
	Short Run	1.1809	1.2467
(2)	Long Run	0.0472	0.9737
	Short Run	1.1937	1.2497
(3)	Long Run	0.0477	0.9717
	Short Run	1.2571	1.2685
(4)	Long Run	0.0488	0.9708
	Short Run	1.2984	1.2774

Notes:

1. Basic CIA model (Cooley and Hansen, 1989).
2. Basic CIA model + Risk-neutral financial intermediary with endogenous default (Ahn *et al.*, 2014), where $(\gamma, \eta_F, \eta_B) = (0, 1, 1)$.
3. Basic CIA model + Risk-averse financial intermediary with endogenous default, where $(\gamma, \eta_F, \eta_B) = (0.25, 1, 1)$.
4. Basic CIA model + Risk-averse financial intermediary with endogenous default and higher degree of sensitivity for a non-pecuniary default penalty, where $(\gamma, \eta_F, \eta_B) = (0.25, 1, 1.35)$.