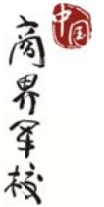


Working Paper No. 2014003



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Competition in Multi-Echelon Distributive Supply Chains with Linear Demand

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Competition in Multi-Echelon Distributive Supply Chains with Linear Demand

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Abstract

We study competition in multi-echelon supply chains with a distributive structure. Firms in the supply chain are grouped into homogeneous sectors that contain identical firms with identical production capabilities that all produce exactly one undifferentiated product. Each sector may distribute its product to several different downstream sectors, and each sector is only supplied by a single upstream sector. The demand curves in final markets are assumed to be linear, as are the variable costs of production in all sectors. Competition is modeled via the Successive Cournot model in which firms choose production quantities for their downstream market so as to maximize their own profits, given prices for the input. Under these assumptions, equilibrium prices, quantities, and firm-level profits for any multi-echelon distributive network can be derived. We discuss the network transformation properties, and by using these properties, we examine the effect of demand parameter changes and cost changes on any firm's equilibrium price, quantity, and profit. We also explore the effects of entry on the equilibrium solution. While the effects of upstream entry on downstream sectors are as expected, the effect of downstream entry on upstream sectors, and therefore on sectors in parallel (lateral) paths can be quite counter-intuitive.

Keywords: Distributed decision making; Successive Cournot competition; Supply chain management

1 Introduction

In many industries it is common to see multi-stage supply chains, in which different companies occupy different stages of the chain. The number of entrants at any level of a supply chain can vary quite substantially. Supply chains also exhibit considerable structural variation across industries. Basic goods and commodity products are often purchased by several distinct sectors. For example, steel is purchased by the automotive industry as well as by the construction industry. Goods can also be sold in more than one geographically distinct marketplace. Such examples display a distributive structure. In other cases, the supply network can have an assembly structure where many inputs may be required to assemble a product. Of course, both of these characteristics can also be present simultaneously.

In this paper we address competition in multi-echelon distributive supply chains, a generalization of the serial chain model of Corbett and Karmarkar (2001). Firms are grouped into sectors where all firms within a sector are identical. Each sector produces exactly one product and uses exactly one input. A product may be purchased by many sectors. The resulting system can be pictured as a network which has a multi-echelon or arborescent structure in which each sector is represented by a node. Each node is supplied by a unique node, but may be a supplier to more than one node. The network representation of such distributive structure is sketched in Figure 1.

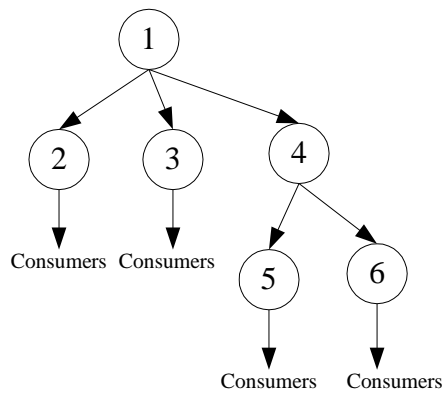


Figure 1: An Example of Distributive Network

Examples of distributive network structures abound. Perhaps the most obvious case is that of simple geographical distribution, where goods from a plant are sold in more than one region, with no cross-selling across regions (due to distance and cost). Distributive structures also occur in many process industries, where one product is used in more than one distinct “downstream” production process to make different products. Examples include metals (steel, aluminum, copper), agricultural products (milk, wheat, corn), petro-chemicals (leading to a vast range of sectors including polymer plastics like

PVC and HDPE, fibers like polyester and chemicals like methanol, ethylene and olefins), and electronic components (like memory chips) going into different boards and products.

Cement production provides a good example of a two-tier distributive network. The first stage is the production of clinker from limestone. At a second tier, clinker is ground, blended with fillers like slag, and packaged into the final product. The grinding plants tend to be closer to market regions and so are geographically distributed, while clinkering must be done close to the sources of raw material (limestone and coal). Grinding plant costs are an order of magnitude smaller than clinker plants, so that there can be many more entrants at the lower tier than at the upper. In some regions, a third tier in this sector is distribution to retail locations, which happens in countries with many small contractors who buy cement locally.

Changes in costs, concentration, technology, network structure, or demand can have both vertical and lateral effects in distributive systems. For example, in the cement production, concentration decrease in one region due to entry of grinding plants might affect the price of clinker, and thus affect the production and profitability of grinding plants in another region, and the price of cement in that region. As another example, corn is used for producing gasohol, as well as starch, corn syrup, and hundreds of other foods, cosmetics and industrial products. An increased demand for gasohol might increase price and production of corn, and thus affect the behaviors of firms in sectors that is not directly linked with the gasohol industry. In a distributive network, the magnitudes of the changes in any sector depend on many, perhaps all, other sectors. Methods for understanding and estimating these changes, and for solving large problems, are not readily available. If multiple concurrent changes occur in these systems, it is not easy to assess the composite effects of those changes. And in some cases, the changes can be counter-intuitive even in terms of direction.

Our purpose is to study system-wide equilibrium behavior while considering vertical and horizontal interactions among sectors in the whole network as well as the competition between firms within each sector (node). We develop models of competition where each individual firm in each sector of a large multi-echelon distributive network acts as a decision maker optimizing its own profit. This model allows us to examine the impact of cost structure, network (distribution) structure and sector concentration on prices, quantities and profits. As in Corbett and Karmarkar (2001) we model competition using the Successive Cournot framework. We provide explicit expressions for equilibrium prices, quantities and firm-level profits. We construct network transformation methods that can compress networks to simpler forms, or expand them to particular (binary) forms. By using these transformation methods, we are able to analyze the impact of cost and demand parameters without having to deal with the complexity or

specific form of the network structure. Importantly, we are also able to study the effect concentration in any sector in the network on any other sector. These effects are in some cases quite non-intuitive.

Conventional wisdom and earlier studies in the literature state that lower concentration (a larger number of firms) in a market generally causes the total output of that market to increase, the consumer price to decrease, and each incumbent's profit to decrease (Seade 1980a). These results are indeed obtained in the serial and assembly cases, under assumptions similar to those of this paper. However, we find that for the distributive case, with the consideration of vertical and horizontal interactions, firm-level profit for incumbents could increase or decrease with changes in concentration. This result is due to the combined effect of competition and upstream resource price changes.

Although this paper adopts the same Successive Cournot Model to analyze the distributive structure as Corbett and Karmarkar (2001) for the serial case and Carr and Karmarkar (2005) for the assembly case, the equilibrium solution provides different insights from those existing papers. In the distributive network, contrary to traditional wisdom, when sector concentration changes, the resource price could increase, decrease, or remains the same depending on the relative parameters of all the distributive sectors. Thus, profits and production of the incumbents could change in either direction, under the combined effects of resource prices and competition. This unique structure behavior is not seen in the serial and assembly structures analyzed in the previous papers. Moreover, different from the serial case, vertical integration for distributive case can make the total profit of the integrator increase or decrease depending on the concentration level and cost parameters.

We note that the (post-entry competition) model of this paper is a pre-requisite not only for entry decisions, but also for other types of analyses. For example, a facility location decision in the context of a supply chain could use the post-entry production model to understand the consequences of alternative location choices. Ho et al. (2004) study competitive location with a simpler imbedded one-tier Cournot model of production competition after location choice. But in many sectors, for example cement, petrochemicals, or food processing, there are multiple tiers involved. This more complex and realistic location problem remains to be investigated. As another example, consider the consequences of process improvement at some stage in an industry to reduce variable costs. The results of such changes can ripple to upstream, downstream and lateral sectors. The tools to analyze such changes do not exist today. Vertical integration decisions can be analyzed in a manner similar to the example in Corbett and Karmarkar (2001) for the serial chain case. In all these examples, the fast solution of the competitive model is a necessary pre-requisite.

The distributive model can also be combined with the assembly model of Carr and Karmarkar (2004) to study more complex production networks. However, this extension is not trivial, and the solution

approach may not be general, since it can depend on the specific structure of the network being considered.

2 Literature

Our analysis is in the tradition of the Successive Cournot oligopoly literature, though most previous research has been limited to the two tier serial case with only one or two entrants at each tier. In fact, the upstream tier is often taken to consist of a single (monopolist) firm. Furthermore, most of the existing literature is directed towards policy issues related to vertical integration and market foreclosure, often in a setting with the upstream monopolist integrating forward. Our approach is directed at the modeling, analysis and solution of general networks, in order to understand the implications for production decisions (quantities), the resulting prices, and the effects of changes in network structure, variable cost structure across the network, end demand, and concentration at network stages.

Machlup and Taber (1960) present an early discussion of successive oligopoly and vertical integration. Greenhut and Ohta (1979) and Abiru (1988) show that vertical integration by a monopolist in the supplying sector, by and large leads to higher outputs and lower prices. The seeming paradox here is that a monopolist integrating forward, can drive out competitors from downstream markets, and yet social welfare can be increased. Essentially, this happens because vertical integration avoids double marginalization. Quan and Rogers (2004), follow an approach similar to Corbett and Karmarkar (2004) to examine a two-tier network of a telecommunications firm purchasing software tools from an upstream vendor; the firm then employs the tools to produce both a product (programmed queries) and services.

Tyagi (1999) studies the effects of downstream entry in a two tier serial setting when the upstream tier consists of a monopolist and the downstream tier consists of identical firms. He finds that downstream entry could affect the (upstream) price charged to the downstream firms. Depending on the consumer demand functions, this change of upstream price could have negative or positive effect on the profits of downstream incumbents. This effect occurs for certain demand conditions, and cannot occur for linear demand in a serial chain. In this paper we show that it can occur in distributive chains even when the end market demand is linear, due to an entirely different mechanism that has to do with the distributive structure rather than with the shape of the end market demand curve.

Other related economic literature includes Ziss's (1995) study of horizontal mergers within a setting of two tiers with two entrants in each tier, Vickers's (1995) study of regulation in serial chain competition and Seade's (1980) study on the effects of concentration and entry. As we have mentioned, none of these papers considers a network where there are multiple tiers of distributors or manufacturers, or a distributive structure.

Our paper is closest in methodology and spirit to Corbett and Karmarkar (2001) who study multi-tier serial supply chains. Also closely related is the paper by Carr and Karmarkar (2005) that analyzes multi-echelon assembly networks. The distributive and assembly settings are complementary generalizations of the serial case. There are significant differences in the two generalizations that derive from the different underlying network structures. In the assembly case, a key advance was in representing how quantity matching takes place across supplying sectors (corresponding to bill-of-materials relationships). In the distributive case, an important insight is that upstream firms have to strike a balance between multiple downstream sectors when discrimination is not possible. This compromise then has implications for the consequences of changes in parameters or structure.

The ability to analyze the equilibrium behavior of general distributive networks distinguishes our paper from previous work, and as far as we are aware, this is the first paper to address this setting.

3 The Two-tier Distributive Model

In the two-tier case, there is a single upstream sector (consisting of several homogenous firms) and multiple sectors in the lower tier (each with several firms) as shown in Figure 2.

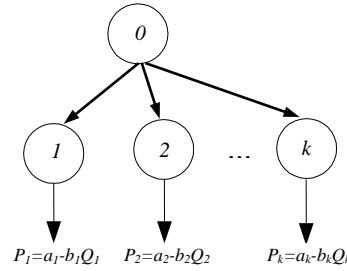


Figure 2: A Two-tier Case

Firms at each downstream sector i face an independent consumer market, where the price and the aggregate production quantities are linearly related by $p_i = a_i - b_i Q_i$. Here, a_i is the market reservation price, the supremum price at which demand will be positive. The other parameters b_i is price sensitivity of the market. We also define $B_i := 1/b_i$ for convenience.

We use the following notation:

v_i , the variable cost of production and distribution in sector i , $i=0, 1, \dots, k$.

n_i , (a positive integer), the number of firms in sector i . For convenience, we also use a parameter N_i defined by $N_i := n_i / (n_i + 1)$.

p_i , the common price charged by all firms in sector i to all downstream sectors.

q_i , the quantity chosen by a single firm in sector i . As in a standard one sector Cournot model, all firms within a sector produce the same quantity at equilibrium, so the individual firms' production quantities can (at equilibrium) be expressed as $q_i = Q_i/n_i$.

π_i , the profit of a single firm in sector i .

We assume that firms in each lower tier sector compete in the Cournot sense, and competition in the network follows the Successive Cournot framework (Machlup and Taber, 1960). That is to say, each firm in each lower tier sector chooses production quantity to maximize its profit, given a demand curve and a price for the input supplied by the upstream sector. For the upstream sector the aggregate quantity across all downstream (lower tier) sectors as a function of the resource price, establishes a demand curve for the resource supplying (upstream) sector. Firms in that sector compete in the Cournot sense by choosing quantities given this demand curve. Costs of resources or inputs at this sector are assumed to be exogenous to the model.

We adopt the equilibrium criteria defined by Carr and Karmarkar (2005). For each sector:

1. Given the demand curve faced by the sector and the resource price charged by the upstream sector, no firm in the sector has an incentive to unilaterally deviate from its production quantity.
2. The aggregate quantity produced in every sector is balanced with the aggregate quantity of the required resource (i.e., markets clear).

The optimal production quantities and equilibrium prices for this system are as follows.

Proposition 1 *The equilibrium prices, production quantities, and profits for the two-tier distributive network in figure 2 are*

$$\begin{aligned}
 p_0 &= (1 - N_0)a_0 + N_0v_0 \\
 Q_0 &= B_0(a_0 - p_0) = N_0B_0(a_0 - v_0) \\
 \pi_0 &= (1 - N_0)^2 Q_0^2 / (B_0 N_0^2) = B_0(1 - N_0)^2(a_0 - v_0)^2 \\
 p_i &= N_i[(1 - N_0)a_0 + N_0v_0] + [(1 - N_i)a_i + N_iv_i], \quad i = 1 \dots k \\
 Q_i &= B_i(a_i - p_i) = N_iB_i[a_i - v_i - v_0 - (1 - N_0)(a_0 - v_0)], \quad i = 1 \dots k \\
 \pi_i &= \frac{(1 - N_i)^2}{B_i N_i^2} Q_i^2 = (1 - N_i)^2 B_i[a_i - v_i - v_0 - (1 - N_0)(a_0 - v_0)]^2, \quad i = 1 \dots k
 \end{aligned}$$

where $a_0 := \sum_{i=1..k} N_i B_i (a_i - v_i) / \sum_{i=1..k} N_i B_i$ and $B_0 := \sum_{i=1..k} N_i B_i$.

The proof is given in the Appendix. An important property of this proposition is that the equilibrium price for the resource supplied by the upstream tier is related to the prices that would be charged in the perfect discrimination case, in a specific way. Suppose that perfect discrimination were possible, i.e.

that in the Cournot setting the upper tier could determine the quantities supplied to each downstream sector (node) independently. The system can then be decomposed into k separate serial systems. Let p_{0j} be the resource price in the j 'th such system. Then we have

Corollary 1 *The resource price p_0 is a convex combination of the perfect discrimination prices p_{0j} , $j=1, \dots, k$:*

$$p_0 = \sum_{j=1..k} N_j B_j p_{0j} / \sum_{i=1..k} N_i B_i . \quad (1)$$

The proof is given in the Appendix. The qualitative insight from this result is that the resource price in the distributive case is a compromise between the prices that could have been obtained from each downstream purchasing sector, under perfect discrimination. So for example, the entry of another potential buyer of the resource will create a price shift, which could be in either direction, depending on the price and the weighting given to the new sector. Note that the weights in the convex combination term depend on the N s which are measures of the concentration of a sector, and on the B s which are a measure of the price sensitivity of the imputed demand curve of the sector. This result reveals the underlying forces of interaction among downstream sectors when affecting the resource price. However, it deserves to be mentioned that the detailed form of convex combination is highly related to the demand linearity, without which it would be hard to derive a simple form of decomposition.

4 The Multi-tier Distributive Network and Structural Properties

We now consider general distributive supply networks as exemplified by Figure 1. We restrict attention to situations in which the aggregate production quantity along each arc of a given network is strictly positive at equilibrium. This assumption can be formally expressed as equilibrium condition, which, together with the demand linearity, guarantees the uniqueness of the equilibrium solution.

The arborescent structure of distributive networks implies that each network has a single root sector. We label the sectors of network top down with the root sector as sector 1. For each sector i (node i), we define:

D_i as the set of all the sectors downstream of sector i . For example, in figure 1, $D_1 = \{2, 3, 4, 5, 6\}$ and $D_4 = \{5, 6\}$.

S_i as the set of sectors immediately downstream of sector i . In figure 1, $S_1 = \{2, 3, 4\}$.

$\varphi_{i,s}$ as the set of sectors along (i, s) , the path from sector i to sector s . In figure 1, path $(1, 5)$ is the path from sector 1, through sector 4, to sector 5; and $\varphi_{1,5} = \{1, 4, 5\}$.

Proposition 2 *For a given distributive network, the derived demand curve for any upstream sector i is*

$$p_i = a_i - b_i Q_i, \text{ or } Q_i = B_i(a_i - p_i), \quad (2)$$

where the parameters B_i , a_i , and b_i are computed iteratively tier by tier as

$$B_i = \sum_{s \in S_i} N_s B_s, \quad b_i = \frac{1}{\sum_{s \in S_i} N_s B_s}, \quad \text{and} \quad a_i = \frac{\sum_{s \in S_i} N_s B_s (a_s - v_s)}{\sum_{s \in S_i} N_s B_s}.$$

The proof is similar to the proof for Proposition 1 and is omitted here. The expressions in (2) provide some interesting properties of the derived demand curves seen by upstream sectors. As in the serial supply chain case (Corbett and Karmarkar, 2001), B decreases going upstream. That is to say, the quantities demanded at upstream sectors are less sensitive to upstream price than sectors which are closer to the markets. Note also that the reservation price a_i seen by a tier i is equal to the weighted average of the reservation prices along i 's downstream arcs, $a_s - v_s, s \in S_i$. In the extreme case when $a_s - v_s$ is the same along each branch, a_i equals $a_s - v_s$. In such situations, a_i does not change with B_s or N_s . This property is crucial for our later discussion of the effect of downstream entry to the prices and profits in upstream sectors.

Using Proposition 2, we can iteratively derive the demand curve for each sector, starting from the sectors facing final consumers. From these curves, the equilibrium price condition for each sector can be derived. All the price conditions together comprise a system of independent linear equations, which is solvable. The equilibrium quantities are then derived by substituting the prices back into demand curves. This solution method can be conveniently expressed in matrix notation, as given in the following proposition.

Proposition 3 *Equilibrium prices of sectors in a distributive network are the solution to*

$$T \cdot \bar{p} = R$$

where \bar{p} is the vector of prices (one element per sector), R is a column vector (one element per sector) with each element $R_i = (1 - N_i)a_i + N_i v_i$, and T is a lower triangular matrix, populated with element

$$T_{ij} = \begin{cases} 1 & \text{if } i = j \\ -N_i & \text{if } i \in S_j \\ 0 & \text{otherwise} \end{cases}.$$

The proof is omitted as it follows from the above discussion. Note that T can be inverted to give T^{-1} with elements

$$T_{ij}^{-1} = \begin{cases} 1 & \text{if } i = j \\ N_{j,i} / N_j & \text{if } i \in D_j \\ 0 & \text{otherwise.} \end{cases}$$

where $N_{j,i}$ is defined as $N_{j,i} := \prod_{s \in \varphi_{j,i}} N_s$. Applying T^{-1} gives the following result.

Proposition 4 *At equilibrium, the price of sector i in a distributive network is*

$$p_i = \sum_{j \in \varphi_{1,i}} (N_{j,i} / N_j) [(1 - N_j)a_j + N_j v_j], \quad (3)$$

where $N_{j,i}$ is defined as $N_{j,i} := \prod_{s \in \varphi_{j,i}} N_s$.

T is a structure matrix that captures the relationship between the structural features of a distributive network (i.e., the sector connections, concentrations, and demand functions) and the equilibrium competition prices. This proposition provides an explicit form for the equilibrium prices for a given network. Equilibrium quantities and profits can then be derived accordingly. Note that although the price of sector i , p_i in (3) depends only on parameters of sectors on the upstream of sector i , those parameters are derived from parameters of all their downstream sectors, as shown in Proposition 2. Therefore, the equilibrium price of one sector i , p_i , is the result of interaction of all sectors in the whole network.

The main purpose of the current research is to analyze the equilibrium changes as a result of cost parameters and sector concentration changes for any given distributive network. This goal, however, cannot be easily reached by directly taking first order differentiation of the equilibrium results derived from Proposition 3 and 4, due to the intertwine of parameters across a general complex distributive network. We now construct network transformation methods using some structural properties of the distributive network. Through these transformation methods, we can discuss the comparative statics with cost and sector concentration changes without having to deal with the general structure any more.

The structural properties are quite similar to the assembly network discussed in Carr and Karmarkar (2005). As in Carr and Karmarkar (2005), we regard two sub-networks or two sets of nodes as equivalent with respect to the rest of the network if, after substituting one network (set of nodes) for the other, the rest of the network has the same equilibrium prices, quantities, and firm-level profits. Noting that a subnetwork essentially communicates to the rest of the

network through the demand curve that is provided to the subnetwork's root node, this means that two sub-networks are equivalent when they show the same demand curve to the rest of the network.

Using this concept, distributive network has the features of expandable and compressible, similar to the assemble network shown in Carr and Karmarkar (2005). As illustrated by Figure 3, a network (a) can be expanded to a network (b) without affect any existing sectors' equilibrium results. Sector D is a dummy sector with $N_d = 1$ and $v_d = 0$. Figure 4 shows that sector 1 firms are unaffected if sector 2 and 3 are compressed into a simple demand curve.

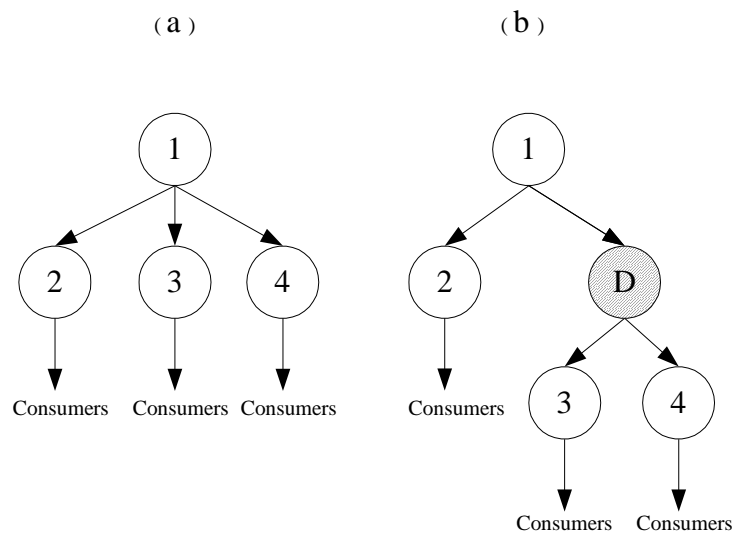


Figure 3: Network Expandability

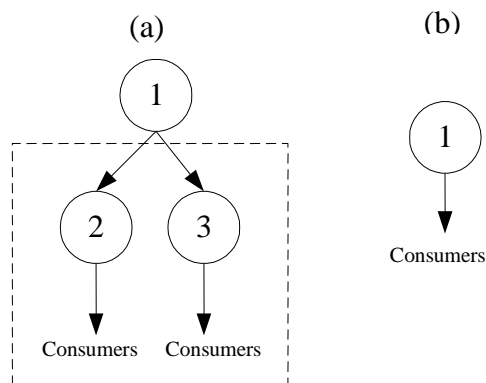


Figure 4: Network Compressibility

Turning to comparative statics, we now consider the sensitivity of a sector's equilibrium prices, profits, and quantities to other sectors' parameters where i will be the sector at which a change occurs, and we consider how this affects another sector s .

Proposition 5 *Suppose s is any sector not upstream of sector i , and let u be the first node encountered that is upstream of both i and s . Then, a parameter change at i is completely communicated to sector s through p_u . That is, at equilibrium, p_s increases iff p_u increases (as a result of the change at i) and, equivalently, π_s decreases iff p_u increases.*

The proof is omitted. Using this proposition together with compressibility and expansibility allows us to analyze the effect of parametric changes very simply. We only need to examine the sectors along the path $(1, i)$ to discuss the effects of parameter changes in sector i . Using the compressibility property, the rest of the network can be compressed to single sectors without changing the equilibrium solution to the network. Therefore, the general multi-tier distributive network can be compressed to a simple binary tree as shown in Figure 5(b). In the following discussion, we focus on the equilibrium prices, quantities, and profits of the sectors along the path $(1, i)$ in the binary structure shown in Figure 5(b). The other sectors are distinguished by a prime ($'$). In the figure, we assume the market parameters (a s and B s) of the leaf sectors (sector $i', i-1', \dots, 2'$ and sector i) are given.

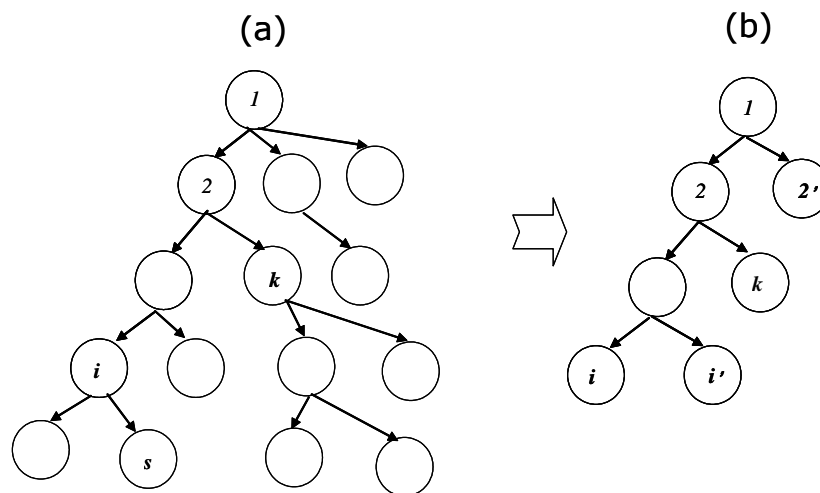


Figure 5: Convert a Distributive Network into a Binary Tree

The following proposition shows how v_i influences equilibrium prices and profits in each sector along the path $(1, i)$.

Proposition 6 (Illustrated by Figure 5(b).) *Suppose that v_i , the variable production cost in sector i , increases. At equilibrium:*

- 1) p_i increases, Q_i and π_i decrease.
- 2) If sector j is upstream of i ($j \in \varphi_{1,i}$), then p_j , Q_j , and π_j decrease.
- 3) If sector j is downstream of i then p_j increases; Q_j and π_j decrease.
- 4) Otherwise (i.e., j is a lateral sector that links with sector i through the same resource sector) p_j decreases; Q_j and π_j increase.

The effects of changes in v_i are intuitive in that increased production cost for firms in a sector increases selling prices and lowers profits. It also decreases the prices and firm-level profits of upstream sectors in the whole supply chain, which means the cost increase is passed along the supply chain eventually resulting in lower prices, lower production, and lower profit margins for all upstream sectors. Interestingly, however, for the lateral sectors in the network that are not upstream nor downstream of sector i , cost increase in sector i causes the resource price of the connecting node to decrease, and thus causes the firms in those sectors to be more profitable.

Proposition 7 *Suppose that a_i increases (at a sector i supplying a consumer market). At equilibrium:*

- 1) Q_i , p_i , and π_i all increase.
- 2) If sector j is upstream of i then Q_j , p_j , and π_j all increase.
- 3) If sector j is not upstream of i then p_j increases, Q_j and π_j decrease.

So, increased profit margins can be passed upstream along the chain, causing prices to increase in resource markets and profits to increase in upstream firms. However, to firms in the "substitute channels" of the network, an increase in a_i has negative effects: their prices increase and profits decrease as a result of the change.

5. The Effects of Sector Concentration

The dependence of equilibrium outputs, prices and profits on industry concentration is a fundamental issue in economic analysis. Conventional wisdom holds that with lower concentration (more firms), industry price ought to decline and per firm output and profit ought to decrease. In single tier Cournot competition, this claim holds for most general

demand conditions. In multi-tier networks, we might expect that lower concentration in a lower tier would lead to higher market power and higher profits in an upper (supplying) tier. However, these expectations may be violated in distributive networks. The following discussion shows that even with linear demand there exists a range of demand conditions for which an increase in the number of firms in a sector (lower concentration) increases the profits of the sector's existing firms and decreases the profits of upstream firms. This surprising result occurs when entry decreases the potential upstream market, and causes the upstream price to decrease; this then permits the profit margin of the downstream firms to increase. If the effect of upstream price outweighs the effect of increased competition due to the entry, the incumbents' profits in the sector with entry can go up. We note that this seemingly perverse effect is not necessarily a common phenomenon. It occurs for certain parameter ranges, and can disappear with further entry. However, what it underlines is that the distributive structure has characteristics which are specific to that structure, and which lead to consequences not seen in the pure serial and assembly cases.

In the following discussion of entry effects, we start with two-tier case to derive explicit results. Then, we extend the analysis to the multi-tier case and show that certain properties generalize simply while other effects can be more complex. The following proposition summarizes the effects of sector concentration on equilibrium prices, outputs, and profits for two-tier networks.

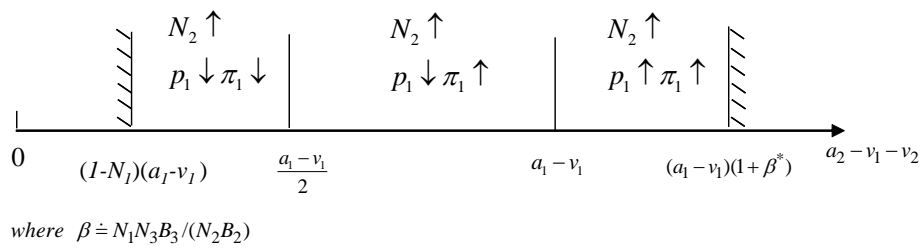


Figure 6: Effect of Increases in N_2 on p_1 and π_1

Proposition 8 *The effects of concentration changes (entry or exit) in the two-tier network of Figure 4(a) are:*

1) *An increase (decrease) in n_1 and N_1 causes p_1 , p_2 , π_1 to decrease (increase), and causes Q_1 , Q_2 , π_2 to increase (decrease).*

2) An increase (decrease) of N_2 due to entry (exit) in that downstream sector causes p_2 to decrease (increase).

3) However, with an increase of N_2 , p_1 and π_1 could increase, decrease or remain constant depending on the relative demand parameters (as shown in Figure 6):

For p_1 : if $a_2 - v_2 > a_1$ (i.e., $a_2 - v_2 > a_3 - v_3$), p_1 increases with N_2 ; if $a_2 - v_2 = a_1$ (i.e., $a_2 - v_2 = a_3 - v_3$), p_1 remains constant; if $a_2 - v_2 < a_1$ (i.e., $a_2 - v_2 < a_3 - v_3$), p_1 decreases with N_2 .

For π_1 : if $a_2 - v_2 - v_1 > (a_1 - v_1)/2$, π_1 increases with N_2 ; if $a_2 - v_2 - v_1 = (a_1 - v_1)/2$, π_1 remains constant; if $a_2 - v_2 - v_1 < (a_1 - v_1)/2$, π_1 decreases with N_2 . $a_2 - v_2 - v_1$ is always greater than $(1 - N_1)(a_1 - v_1)$ and smaller than $(a_1 - v_1)(1 + N_1 N_3 B_3 / (N_2 B_2))$ due to the regularity condition.

4) Moreover, π_2 could increase, decrease, or remain constant with the increase of N_2 . Specifically

$$\text{if } a_2 - v_2 - v_1 < \frac{B_1 + (1 - N_2)B_2}{B_1 + (1 - N_2)B_2 - N_1(1 - N_2)B_2} (1 - N_1)(a_1 - v_1), \quad (4)$$

π_2 increases with N_2 . Otherwise, π_2 decreases with N_2 (or remains constant when the above expression holds with equality).

Parts 1) and 2) of Proposition 8 state that lower concentration or an increased number of firms in a sector, leads to lower prices in the sector. More interesting and quite counter-intuitive findings are the effects of N_2 on p_1 , π_1 , and π_2 as stated in parts 3) and 4) of Proposition 8. Contrary to conventional wisdom, reduced concentration (entry) in a downstream sector can cause the resource price charged by the upstream suppliers to go up under certain demand and cost conditions. Moreover, for a certain range of demand parameters, the profit of upstream suppliers could go down as a result of the downstream entry. Similarly, incumbents in the same sector (sector 2 in our analysis) could see profits increase.

The last phenomenon is not pervasive. It only happens when the effect of resource price decrease overwhelms the effect of competition. Furthermore, notice that the right hand side of the inequality condition (4) monotonically decreases in N_2 . With continued entry in sector 2, the anomalous effect goes away. However, it illustrates a characteristic of distributive systems. The underlying reason for these effects is the nature of upstream resource price, which as shown in Proposition 2, is a compromise between the resource prices that would be seen with perfect discrimination. The weight of the balance is controlled by the concentration (N_i). For a sector in which the resource price under perfect discrimination is higher, entry intensifies the weight, and makes the equilibrium resource price higher, and vice versa.

To examine the effect of concentration changes at a sector i in a multi-tier network, we only need to focus on the behavior of sectors along the path $(1, i)$. We note that, the reservation prices for upstream sectors a_j ($1 \leq j \leq i$) play a key role in the changes of prices, outputs and firm-level profits. In the following proposition, we summarize the direction of change of a_j with respect to N_i .

Proposition 9 *The direction of change of the market reservation price a_j at sector j , $j \in \varphi_{1,i}$, with respect to N_i can be captured by its first derivative as*

$$\frac{\partial a_j}{\partial N_i} = \frac{N_{j+1,i-1} B_i}{B_j} (a_i - v_{j+1,i} - a_j)$$

where $N_{j+1,i-1} = \prod_{s=j+1}^{i-1} N_s$ (if $j=i-1$, $N_{j+1,i-1} = 1$) and $v_{j+1,i} = \sum_{s=j+1}^i v_s$.

Thus the reservation price for an upstream sector increases, decreases, or remains the same depending on the relative value of the reservation prices along the channel where entry occurs and the reservation price of the market, which is the weighted average value of each downstream channel as discussed earlier. This result, which is different from the cases in serial chains (Corbett and Karmarkar, 2001) or assembly networks (Carr and Karmarkar, 2005), enables us to examine the price changes of any sector with the change of N_i .

Proposition 10 *In a multi-echelon distributive network, the price at tier i , p_i , decreases monotonically with N_i .*

Using (3), p_j , the price charged by the upstream sectors j , can be expressed as

$$p_j = \sum_{k=1..j} N_{k+1,j} [(1 - N_k) a_k + N_k v_k]$$

Since each individual " a " is a function of N_i , the directive of p_j to N_i can be expressed as

$$\begin{aligned} \frac{\partial p_j}{\partial N_i} &= \sum_{k=1..j} N_{k+1,j} (1 - N_k) \frac{\partial a_k}{\partial N_i} \\ &= \sum_{j=1..j} N_{k+1,j} (1 - N_j) \frac{N_{k+1,i-1} B_i}{B_k} (a_i - v_{k+1,i} - a_k). \end{aligned}$$

Next, profit of sector j can be expressed as

$$\pi_j = B_j (1 - N_j)^2 (a_j - v_j - p_{j-1})^2,$$

and the first differentiation then gives

$$\frac{\partial \pi_j}{\partial N_i} = 2B_j(1-N_j)^2(a_j - v_j - p_{j-1})\left(\frac{\partial a_j}{\partial N_i} - \frac{\partial p_{j-1}}{\partial N_i}\right).$$

We can thus see that upstream market price and profit could decrease, increase or remain constant with downstream entry. Furthermore, the trends in these strategic variables can be even more complicated since they depend on the changes of any upper tier's reservation price and equilibrium price. For example, a sector's price could increase even when the sector's reservation price decreases.

6. Vertical Integration in Distributive Networks

Vertical integration abounds in industries with distributive structure. It is commonly seen in cement industry that clinker plants supply both grinding plants that are integrated with them, and independent grinding plants. Most of the PC manufacturers, such as Dell and HP, bundle monitors with their computers, as well as selling them separately through retail channels.

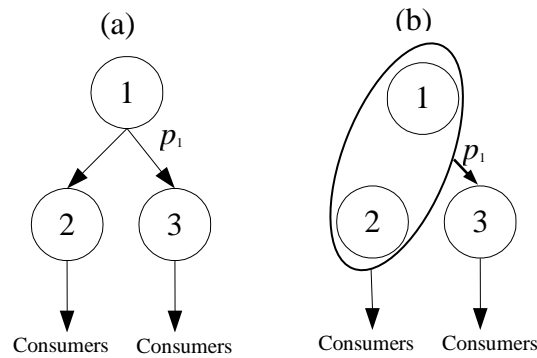


Figure 7: Vertical Integration

The explicit form of equilibrium solution for the distributive networks shown in Proposition 2, 3, and 4 makes it possible to examine the effect of vertical integration in the post-entry game. Corbett and Karmarkar (2001) have examined vertical integration in two tier serial networks, assuming that the numbers of firms in both tiers are the same, to permit comparison of the integrated and un-integrated cases. They find that when each tier has a single firm (monopoly), integration results in higher profits. However, when there are two or more firms in each tier, then the total profits of the network decline.

In the distributive case, there are many more structural alternatives that might be considered with respect to integration – too many to really consider all. However, as in the serial case, the modeling approach developed here allows for any specific case to be analyzed. What is more, the distributive structure leads to phenomena which do not occur in the serial case. Consider a sector that supplies two downstream sectors (Figure 7). We can then have a situation where the upstream sector might integrate forward with one of the downstream sectors but not the other. As in the serial case, we can look at what happens relative to that downstream market. However, here there will also be lateral effects on the other downstream sector. Recalling the result of proposition 2, one can see that after integration, the upstream sector will no longer have to balance the downstream sectors in its pricing decisions, and will take different action with respect to the second (unintegrated) sector. From the point of profitability, the upstream sector will see two sources of profit changes: that from integrating forward, and that from changing its actions with respect to the second downstream sector. In turn, the second un-integrated downstream sector may see either an increase or a decrease in resource price and therefore its profits could either go up or down. The latter (lateral) effect is of course a characteristic of distributive network structure.

Proposition 11 *Assume there are same number of firms in sector 1 and 2 in the two-tier, three-sector case (as in Figure 7). Vertical integration of sector 1 and sector 2 always causes Q_2 to increase. Moreover, if $a_2 - v_2 > a_3 - v_3$, the resource price p_1 decreases; if $a_2 - v_2 = a_3 - v_3$, p_1 remains unchanged; and if $a_2 - v_2 < a_3 - v_3$, p_1 increases. Q_3 and π_3 increase (decrease) iff p_1 increases (decreases). Finally, the total profit of the integrated firms, $\pi_1 + \pi_2$, increases when $n_2 = 1$; otherwise, when $n_2 > 1$, $\pi_1 + \pi_2$ could increase or decrease, depending on the relative value of $a_2 - v_2$ and $a_3 - v_3$.*

In the case where there are more than two downstream sectors, integration of the upper tier with one of the downstream sectors can lead to a wide range of possible outcomes, depending on the specifics of the system.

7. Conclusions

In this paper, we have analyzed competition in pure distributive multi-echelon supply networks, using the Successive Cournot model for oligopolistic competition with multiple tiers. We developed explicit expressions for equilibrium prices and quantities as the solutions to a

set of linear equations that can be derived from the structure of the network. The equilibrium solution is obtained in two steps: 1) Iteratively calculate all the upstream markets' demand parameters; 2) Solve a system of linear equations that involve the demand parameters. We demonstrated certain network transformation principles that allow a network to be compressed or to be expanded to a binary tree structure. These transformations make it straightforward to examine the effects of parametric changes on the equilibrium solution to any distributive network.

Finally, we present some comparative statics results and discuss the effects of entry on equilibrium prices, quantities and firm-level profits. Changes in the variable costs of production have expected effects, as do changes in demand parameters. However, the effects of changes in sector concentration are not as obvious. If the number of firms in a sector increases, the quantity produced in the sector increases and price charged by firms in the sector decreases. Downstream effects are also as expected. However, the upstream consequences are more complicated in that whether upstream prices increase or decrease depends on the demand conditions of the sector where the entry occurs relative to parallel paths. If entry occurs along the channel with less than the average reservation price of the upstream market, the upstream price could decrease rather than increase. With certain demand conditions, the decreased resource price provides a larger profit margin for downstream incumbents and this positive impact on profits can outweigh the competition effect due to entry and thus cause equilibrium profits to increase with entry. Decreased prices in upstream can also cause upstream firms to profit less (although for the two-tier case with a monopoly supplier in the upstream tier, upstream profits always increase with downstream entry). For those firms not along the path between the root sector and the sector with entry, their equilibrium prices, quantities, and profits change according to the change in the price of the connecting node i.e. the sector that connects the firms with the path in question. Thus, we see that some existing intuitions, largely derived either from serial supply chains or from models with a single competitive sector, do not all survive the extension to more complex distributive networks.

The present analysis not only provides results for general distributive supply chains, but also suggests directions for the analysis of other network structures. In ongoing research, we

are investigating the analysis of acyclic multi-echelon networks that have a mix of assembly, distributive, and non-arborescent network topologies. The eventual target of this stream of research is to provide robust techniques to analyze competition in general supply chains and networks with general structures.

References

- Abiru, M. 1988. Vertical integration, variable proportions and successive oligopolies. *Journal of Industrial Economics* 36 315-325.
- Carr, M. S., U. S. Karmarkar. 2005. Competition in multi-echelon assembly supply chains. *Management Science* 51 45-59.
- Corbett, C., U. S. Karmarkar. 2001. Competition and structure in serial supply chains with deterministic demand. *Management Science* 47 966-978.
- Greenhut, M. L., H. Ohta. 1979. Vertical integration of successive oligopolists. *American Economics Review* 69 137--141.
- Machlup, F., M. Taber. 1960. Bilateral monopoly, successive monopoly, and vertical integration. *Economica* 27 101-119.
- Quan, V., S. Rogers. 2004. Modeling a vendor-Telco supply network to deliver two types of telephony services. *Computers and Industrial Engineering* 46 (4) 893-903.
- Rhim, H., T. H. Ho, U. S. Karmarkar. 2003. Competitive location, production, and market selection. *European Journal of Operational Research* 149 211-228.
- Seade, J. 1980. On the effect of entry. *Econometrica*, 48 (2) 479-489.
- Tyagi, R.K.1999. On the effects of downstream entry. *Management Science* 45 59-73
- Vickers, J. 1995. Competition and regulation and vertically related markets. *Review of Economic Study* 62 1-17.
- Ziss, S. 1995. Vertical separation and horizontal mergers. *Journal of Industrial Economics* 43 63-75.

Appendix

Proof of Proposition 1

Starting with sector i of the downstream tier, the first equilibrium criterion means that each firm in the sector selects a production quantity q_i that maximizes its profits given that the firms in the sector

purchase products at a cost of p_0 and incur a variable cost v_i for every unit produced. A single firm thus seeks to maximize revenue of $q_i(p_i - p_0 - v_i)$, where p_i equals $a_i - b_i Q_i$. Differentiation gives us the firm's first order optimality condition which is to select quantity that solves

$$q_i = B_i(a_i - v_i - p_0) / 2 - Q_{-i} / 2, \quad i=1, \dots, k$$

where Q_{-i} is the aggregate quantity produced by all the other firms in sector i . The production decision of each firm in sector i follows this same condition as well due to the identical cost structure of all firms in the sector. This gives us a system of n_i independent linear equations for each sector i . A symmetric solution can be calculated in which every firm produces quantity is

$$q_i = B_i(a_i - v_i - p_0) / (n_i + 1), \quad i=1, \dots, k$$

and the entire sector produces an aggregate quantity,

$$Q_i = n_i q_i = N_i B_i(a_i - v_i - p_0), \quad i=1, \dots, k \quad (\text{A1})$$

We now substitute (A1) into sector i 's demand curve to get the sector i equilibrium price condition

$$p_i = (1 - N_i)a_i + N_i(v_i + p_0), \quad i=1, \dots, k \quad (\text{A2})$$

Next we look at the supplier tier, sector 0. Our second equilibrium criterion requires the supply and demand quantities to be balanced, so the aggregate quantity produced at sector 0 should be equal to the aggregate equilibrium quantities in each of its distributive downstream sector, i.e., $Q_0 = Q_1 + \dots + Q_k$. Taking in the equilibrium aggregate quantity Q_i from (A1), we can derive the demand curve for sector 0 firms as

$$\begin{aligned} Q_0 &= \sum_{i=1..k} N_i B_i(a_i - v_i - p_0) = \left(\sum_{i=1..k} N_i B_i \right) \left(\frac{\sum_{i=1..k} N_i B_i(a_i - v_i)}{\sum_{i=1..k} N_i B_i} - p_0 \right) \\ &\equiv B_0(a_0 - p_0) \end{aligned}$$

where $a_0 := \frac{\sum_{i=1..k} N_i B_i(a_i - v_i)}{\sum_{i=1..k} N_i B_i}$ and $B_0 := \sum_{i=1..k} N_i B_i$.

Now analyzing the production decisions for sector 0 firms gives the equilibrium production quantities as

$$\begin{aligned} q_0 &= (1 - N_0)B_0(a_0 - v_0), \\ Q_0 &= n_0 q_0 = N_0 B_0(a_0 - v_0). \end{aligned}$$

Also, we can derive the sector 0 equilibrium price as

$$p_0 = (1 - N_0)a_0 + N_0 v_0. \quad (\text{A3})$$

Equations (A2) and (A3) taken together are a system of independent linear equations, and can be easily solved to get the equilibrium prices (p_0 is already the equilibrium price),

$$p_i = N_i[(1 - N_0)a_0 + N_0 v_0] + [(1 - N_i)a_i + N_i v_i], \quad i=1 \dots k.$$

Furthermore, substituting these prices into the relevant demand curves gives the equilibrium aggregate production quantity for each sector. Firm level profits can then be derived by substituting back the optimal prices and quantities. Q.E.D.

Proof of Proposition 2

From proposition 1, $p_{0j} = (1 - N_0)a_{0j} + N_0v_0$, where $a_{0j} := a_j - v_j$. Thus, (1) can be derived.

Proof of Proposition 6

We know for the distributive network in figure 5(b), the prices and profits of each individual firm in sector j ($1 \leq j \leq i$) are:

$$\begin{aligned} p_j &= (1 - N_j)a_j + N_j(v_j + p_{j-1}), \\ Q_j &= N_j B_j (a_j - v_j - p_{j-1}), \\ \pi_j &= (1 - N_j)^2 B_j (a_j - v_j - p_{j-1})^2 \end{aligned}$$

Moreover, it is straightforward to derive the following expression,

$$\partial a_j / \partial v_i = -N_{j+1,i} B_i / B_j, \quad \text{for } j \in [1, i-1].$$

To prove 1), note that

$$\begin{aligned} \partial p_i / \partial v_i &= N_i (1 + \partial p_{i-1} / \partial v_i), \\ Q_i &= -N_i B_i (1 + \partial p_{i-1} / \partial v_i), \\ \pi_i &= -2(1 - N_i)^2 B_i (a_i - v_i - p_{i-1}) (1 + \partial p_{i-1} / \partial v_i), \end{aligned}$$

we only need to show that $\partial p_{i-1} / \partial v_i > -1$. This can be done by induction. First, for sector 1,

$$\partial p_1 / \partial v_i = -N_{2,i} (1 - N_1) B_i / B_1 > -N_{2,i} (1 - N_1) B_i / B_1 \times N_1 / (1 - N_1) = N_{1,i} (1 - N_1) B_i / B_1 > -1.$$

Assume the condition holds for sector $i-2$. Then, for sector $i-1$, we have

$$\begin{aligned} \partial p_{i-1} / \partial v_i &= (1 - N_{i-1}) (\partial a_{i-1} / \partial v_i) + N_{i-1} (\partial p_{i-2} / \partial v_i) \\ &> -(1 - N_{i-1}) N_i B_i / B_{i-1} - N_{i-1} = -\frac{N_i B_i + N_{i-1} N_i B_i}{N_i B_i + N_{i-1} B_i} > -1. \end{aligned}$$

Thus, (1) is proved.

Now we use induction to establish (2). Starting from sector 1, we can derive the first derivatives with respect to v_i as

$$\begin{aligned} \partial p_1 / \partial v_i &= (1 - N_1) (\partial a_1 / \partial v_i) = -N_{2,i} (1 - N_1) B_i / B_1 < 0, \\ \partial Q_1 / \partial v_i &= B_1 N_1 (\partial a_1 / \partial v_i) = -N_{1,i} B_i < 0 \\ \partial \pi_1 / \partial v_i &= 2(1 - N_1)^2 B_1 (a_1 - v_1) (\partial a_1 / \partial v_i) = -2(1 - N_1)^2 N_{2,i} B_i (a_1 - v_1) < 0 \end{aligned}$$

Assuming these properties hold for sector $j-1$, we have the following expressions for p_j , Q_j , and π_j ,

$$\begin{aligned}\frac{\partial p_j}{\partial v_i} &= (1 - N_j) \frac{\partial a_j}{\partial v_i} + N_j \frac{\partial p_{j-1}}{\partial v_i} < 0 \\ \frac{\partial Q_j}{\partial v_i} &= B_j N_j \left(\frac{\partial a_j}{\partial v_i} - \frac{\partial p_{j-1}}{\partial v_i} \right) = B_j N_j \left[\frac{\partial a_j}{\partial v_i} - (1 - N_{j-1}) \frac{\partial a_{j-1}}{\partial v_i} - N_{j-1} \frac{\partial p_{j-2}}{\partial v_i} \right] \\ &= B_j N_j \left[-\left(\frac{N_{i,j+1} B_i}{B_j} - \frac{N_{i,j} B_i}{B_{j-1}} \right) + N_{j-1} \left(\frac{\partial a_{j-1}}{\partial v_i} - \frac{\partial p_{j-2}}{\partial v_i} \right) \right].\end{aligned}$$

Since $N_{i,j+1} B_i / B_j > N_{i,j} B_i / B_{j-1}$, we have $\partial Q_j / \partial v_i < 0$.

As to the profits of sector j ,

$$\frac{\partial \pi_j}{\partial v_i} = 2B_j (1 - N_j)^2 (a_j - v_j - p_{j-1}) \left(\frac{\partial a_j}{\partial v_i} - \frac{\partial p_{j-1}}{\partial v_i} \right) < 0.$$

Thus (2) is proved. It is straightforward to derive (3) and (4) from (2) and proposition 5. Q.E.D.

Proof of Proposition 7

The proof is analogously similar to proposition 6, and is omitted here.

Proof of Proposition 8

1) For the three-sector two-tier network of figure 4(a), according to proposition 1, the price and profit of each individual firm in sector 1 and 2 are given by:

$$\begin{aligned}p_1 &= (1 - N_1) a_1 + N_1 v_1, \\ p_2 &= (1 - N_2) a_2 + N_2 (p_1 + v_2), \\ Q_1 &= N_1 B_1 (a_1 - v_1), \\ Q_2 &= N_2 B_2 (a_2 - v_2 - p_1), \\ \pi_1 &= b_1 q_1^2 = (1 - N_1)^2 B_1 (a_1 - v_1)^2, \\ \pi_2 &= b_2 q_2^2 = (1 - N_2)^2 B_2 (a_2 - v_2 - p_1)^2.\end{aligned}$$

Treating N_1 as a continuous variable and taking the first derivative of these expressions with respect to N_1 , we can easily show 1).

2) Differentiating p_2 with respect to N_2 , we get

$$\begin{aligned}\frac{\partial p_2}{\partial N_2} &= -(a_2 - v_2 - p_1) + N_2 \frac{\partial p_1}{\partial N_2} \\ &= -[a_2 - v_2 - p_1 - (1 - N_1)(a_1 - v_1)] + (1 - N_1) N_2 B_2 [(a_2 - v_2 - v_1) - (a_1 - v_1)] / B_1 \\ &= -(N_3 B_3 + N_1 N_2 B_2)(a_2 - v_2 - v_1) / B_1 + (1 - N_1) N_3 B_3 (a_1 - v_1) / B_1\end{aligned}$$

Due to the regularity condition, $Q_1 > 0$, $Q_2 > 0$, and $Q_3 > 0$. Taking Q_1 , Q_2 , and Q_3 from

proposition1 and simplifying the expressions, we have

$$\begin{aligned} a_1 - v_1 &> 0 \\ a_2 - v_2 - v_1 &> (1 - N_1)(a_1 - v_1) \\ a_2 - v_2 - v_1 &< [1 + N_1 N_3 B_3 / (N_2 B_2)](a_1 - v_1) \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\partial p_2}{\partial N_2} &< -\frac{N_3 B_3 + N_1 N_2 B_2}{B_1} (1 - N_1)(a_1 - v_1) + \frac{N_3 B_3}{B_1} (1 - N_1)(a_1 - v_1) \\ &= -\frac{N_1 N_2 B_2}{B_1} (1 - N_1)(a_1 - v_1) < 0. \end{aligned}$$

3) To show the effect of N_2 on p_1 , we take the first derivative of p_1 with respect to N_2 as

$$\frac{\partial p_1}{\partial N_2} = (1 - N_1) \frac{\partial a_1}{\partial N_2} = (1 - N_1) \frac{N_3 B_2 B_3}{B_1^2} [(a_2 - v_2) - (a_3 - v_3)].$$

Thus, if $a_2 - v_2 > a_3 - v_3$, $\partial p_1 / \partial N_2 > 0$; $a_2 - v_2 < a_3 - v_3$, $\partial p_1 / \partial N_2 < 0$; $a_2 - v_2 = a_3 - v_3$, $\partial p_1 / \partial N_2 = 0$.

To show the effect of N_2 on π_1 , we have

$$\begin{aligned} \frac{\partial \pi_1}{\partial N_2} &= B_2 (1 - N_1)^2 (a_1 - v_1)^2 + 2B_1 (1 - N_1)^2 (a_1 - v_1) \frac{\partial a_1}{\partial N_2} \\ &= B_2 (1 - N_1)^2 (a_1 - v_1)^2 + 2B_1 (1 - N_1)^2 (a_1 - v_1) \frac{B_2}{B_1} (a_2 - v_2 - a_1) \\ &= B_2 (1 - N_1)^2 (a_1 - v_1) [2(a_2 - v_2 - v_1) - (a_1 - v_1)] \end{aligned}$$

Therefore, if $a_2 - v_2 - v_1 < (a_1 - v_1) / 2$, $\partial \pi_1 / \partial N_2$ is negative, i.e., π_1 decreases with N_2 . If $a_2 - v_2 - v_1 > (a_1 - v_1) / 2$, π_1 increase with N_2 .

4) To show the effect of N_2 on π_2 , we take the first derivative of π_2 with respect to N_2 ,

$$\begin{aligned} \frac{\partial \pi_2}{\partial N_2} &= -2B_2 (1 - N_2)(a_2 - v_2 - p_1)^2 - 2B_2 (1 - N_2)^2 (a_2 - v_2 - p_1) \frac{\partial p_1}{\partial N_2} \\ &= -2B_2 (1 - N_2)(a_2 - v_2 - p_1)X, \end{aligned}$$

where X is defined as

$$\begin{aligned}
X &:= (a_2 - v_2 - p_1) + (1 - N_2)(1 - N_1) \frac{B_2}{B_1} [(a_2 - v_2 - v_1) - (a_1 - v_1)] \\
&= [a_2 - v_2 - v_1 - (1 - N_1)(a_1 - v_1)] + (1 - N_2)(1 - N_1) \frac{B_2}{B_1} [(a_2 - v_2 - v_1) - (a_1 - v_1)] \\
&= [1 + (1 - N_2)(1 - N_1) \frac{B_2}{B_1}] (a_2 - v_2 - v_1) - [1 + (1 - N_2) \frac{B_2}{B_1}] (1 - N_1)(a_1 - v_1).
\end{aligned}$$

Notice that $[1 + (1 - N_2)(1 - N_1)B_2 / B_1]$ is always smaller than $[1 + (1 - N_2)B_2 / B_1]$. Therefore, if $a_2 - v_2 - v_1$ is very close to its lower bound $(1 - N_1)(a_1 - v_1)$, X could be negative, which means $\partial\pi / \partial N_2 > 0$. More specifically,

$$\text{if } a_2 - v_2 - v_1 < \frac{B_1 + (1 - N_2)B_2}{B_1 + (1 - N_2)B_2 - N_1(1 - N_2)B_2} (1 - N_1)(a_1 - v_1),$$

π_2 decreases with N_2 . Otherwise, π_2 increases with N_2 . Q.E.D.

Proof of Proposition 9

Applying proposition 3 iteratively along the path (j, i) gives

$$\begin{aligned}
B_j &= N_{j+1}B_{j+1} + N_{(j+1)'}B_{(j+1)'} = N_{j+1,i}B_i + \sum_{j < s < i} N_{j+1,s}N_{s'}B_{s'} + N_{(j+1)'}B_{(j+1)'}, \\
a_j &= \frac{N_{j+1}B_{j+1}(a_{j+1} - v_{j+1})}{B_j} + \frac{N_{(j+1)'}B_{(j+1)'}(a_{(j+1)'} - v_{(j+1)'})}{B_j} \\
&= \frac{N_{j+1,i}B_i(a_i - v_{j+1,i})}{B_j} + \sum_{j < s < i} \frac{N_{j+1,s}N_{s'}B_{s'}(a_{s'} - v_{j+1,s} - v_{s'})}{B_j} + \frac{N_{(j+1)'}B_{(j+1)'}(a_{(j+1)'} - v_{(j+1)'})}{B_j}.
\end{aligned}$$

where $v_{j+1,s} := \sum_{k=j+1}^s N_k$. In a_j , only $N_{j+1,i}$ and B_j change with the increase of N_i and all the other parameters remain constant. Therefore,

$$\begin{aligned}
\frac{\partial a_j}{\partial N_i} &= \frac{\partial}{\partial N_i} \left(\frac{N_{j+1,i}B_i(a_i - v_{j+1,i})}{B_j} + \sum_{j < s < i} \frac{N_{j+1,s}N_{s'}B_{s'}(a_{s'} - v_{j+1,s} - v_{s'})}{B_j} \right. \\
&\quad \left. + \frac{N_{(j+1)'}B_{(j+1)'}(a_{(j+1)'} - v_{(j+1)'})}{B_j} \right) \\
&= \frac{1}{B_j} (N_{j+1,i-1}B_i(a_i - v_{j+1,i})) + \frac{a_j}{B_j} \frac{\partial}{\partial N_i} (N_{j+1,i}B_i + \sum_{j < s < i} N_{j+1,s}N_{s'}B_{s'} + N_{(j+1)'}B_{(j+1)'}) \\
&= \frac{N_{j+1,i-1}B_i}{B_j} (a_i - v_{j+1,i}) + \frac{a_j}{B_j} (N_{j+1,i-1}B_i) \\
&= \frac{N_{j+1,i-1}B_i}{B_j} (a_i - v_{j+1,i} - a_j)
\end{aligned}$$

where $N_{j+1,i-1} = 1$, if $j = i - 1$. Q.E.D.

Proof of Proposition 10

As in (3), $p_i = \sum_{r=1}^i N_{r+1,i} [(1-N_r)a_r + N_r v_r]$. Note both $N_{r+1,i}$ and a_r change with N_i . Taking first derivative of p_i with respect to N_i and applying (4) give

$$\begin{aligned} \frac{\partial p_i}{\partial N_i} &= \sum_{r=1}^{i-1} [N_{r+1,i-1}(1-N_r)a_r + N_{r,i-1}v_r] - (a_i - v_i) + \sum_{r=1}^{i-1} N_{r+1,i}(1-N_r) \frac{\partial a_r}{\partial N_i} \\ &= \sum_{r=1}^{i-1} [N_{r+1,i-1}(1-N_r)a_r + N_{r,i-1}v_r] - (a_i - v_i) + \\ &\quad + \sum_{r=1}^{i-1} N_{r+1,i}(1-N_r) \frac{N_{r+1,i-1}B_i}{B_r} (a_i - v_{r+1,i} - a_r) \end{aligned}$$

This can be further simplified as

$$\frac{\partial p_i}{\partial N_i} = \sum_{r=1}^{i-1} N_{r+1,i-1}(1-N_r)(a_r - v_{1,r}) - (a_i - v_{1,i}) + \sum_{r=1}^{i-1} (1-N_r) \frac{N_{r+1,i-1}N_i B_i}{B_r} [(a_i - v_{1,i}) - (a_r - v_{1,r})]. \quad (\text{A4})$$

To prove that p_i decreases with N_i , we only need to show that $\partial p_i / \partial N_i$ is always negative. We do this in two steps: 1) we construct a series of upper bounds U_i, U_{i-1}, \dots, U_2 , and show that $\partial p_i / \partial N_i$ is smaller than U_i ; 2) we show that $U_i < U_{i-1} < \dots < U_2 < 0$.

First, we develop a few inequalities used in the relaxation. According to the regularity condition, for a feasible distributive network, Q_1, Q_2, \dots, Q_i are strictly positive. Thus we have

$$\begin{aligned} Q_i &= B_i(1-N_i)(a_i - v_i - p_{i-1}) \\ &= B_i(1-N_i)(a_i - v_i - \sum_{r=1}^{i-1} N_{r+1,i-1}[(1-N_r)a_r + N_r v_r]) \\ &= B_i(1-N_i)[(a_i - v_{1,i}) - \sum_{r=1}^{i-1} N_{r+1,i-1}(1-N_r)(a_r - v_{1,r})] > 0 \end{aligned}$$

which gives us the following inequality,

$$a_i - v_{1,i} > \sum_{r=1}^{i-1} N_{r+1,i-1}(1-N_r)(a_r - v_{1,r}).$$

Similarly,

$$\begin{aligned} a_{i-1} - v_{1,i-1} &> \sum_{r=1}^{i-2} N_{r+1,i-2}(1-N_r)(a_r - v_{1,r}), \\ &\dots \\ a_2 - v_2 - v_1 &> (1-N_1)(a_1 - v_1). \end{aligned} \quad (\text{A5})$$

It deserves to be mentioned that the above inequalities hold only when $a_r - v_{1,r} > 0$, which is true for the distributive network to be feasible.

Moreover, since for any sector $r > 1$, $B_{r-1} = N_r B_r + N_{r'} B_{r'} > N_r B_r$, we can derive the following inequalities:

$$\frac{N_{2,i} B_i}{B_1} < \frac{N_{3,i} B_i}{B_2} < \dots < \frac{N_{i-1,i} B_i}{B_{i-2}} < \frac{N_i B_i}{B_{i-1}} < 1 \quad (\text{A6})$$

1). Using (A5), we relax $\partial p_i / \partial N_i$ by eliminate $a_i - v_{1,i}$ term in (A4) to reach its upper bound U_i . In general, the upper bound is constructed as $\forall j \in [2, i]$,

$$U_j = \sum_{r=1}^{j-1} \left(\sum_{k=1}^{j-1} \frac{N_{k+1,i-1}^2 N_i B_i (1-N_k)}{B_k} - \frac{N_{r+1,j-1} N_{j,i-1}^2 N_i B_i}{B_r} \right) N_{r+1,j-1} (1-N_r) (a_r - v_{1,r}),$$

where we define $N_{k,k+1} = 1$. For $j = i$,

$$U_i = \sum_{r=1}^{i-1} \left(\sum_{k=1}^{i-1} \frac{N_{k+1,i-1}^2 N_i B_i (1-N_k)}{B_k} - \frac{N_{r+1,i-1} N_i B_i}{B_r} \right) N_{r+1,i-1} (1-N_r) (a_r - v_{1,r}).$$

Now, we need to prove $\partial p_i / \partial N_i$ is bounded by U_i . By combining the $a_i - v_{1,i}$ terms in (A4), $\partial p_i / \partial N_i$ can be further expressed as

$$\frac{\partial p_i}{\partial N_i} = [-1 + \sum_{r=1}^{i-1} (1-N_r) \frac{N_{r+1,i-1}^2 N_i B_i}{B_r}] (a_i - v_{1,i}) + \sum_{r=1}^{i-1} N_{r+1,i-1} (1-N_r) \frac{B_r - N_{r+1,i} B_i}{B_r} (a_r - v_{1,r}). \quad (\text{A7})$$

The coefficient of $a_i - v_{1,i}$ in (A7) is

$$\begin{aligned} & -1 + \sum_{r=1}^{i-1} (1-N_r) \frac{N_{r+1,i-1}^2 N_i B_i}{B_r} \\ & = -1 + \frac{N_i B_i}{B_{i-1}} (1-N_{i-1}) + \frac{N_{i-1,i} B_i}{B_{i-2}} N_{i-1} (1-N_{i-2}) + \dots + \frac{N_{2,i} B_i}{B_1} N_{2,i-1} (1-N_1) \\ & < -1 + \frac{N_i B_i}{B_{i-1}} (1-N_{i-1}) + \frac{N_i B_i}{B_{i-1}} N_{i-1} (1-N_{i-2}) + \dots + \frac{N_i B_i}{B_{i-1}} N_{2,i-1} (1-N_1) \\ & = -1 + \frac{N_i B_i}{B_{i-1}} (1-N_{1,i-1}) < -1 + \frac{N_i B_i}{B_{i-1}} < 0, \end{aligned} \quad (\text{A8})$$

where all the inequalities come from (A6). Therefore, we can relax (A7) using (A5) as

$$\begin{aligned} \frac{\partial p_i}{\partial N_i} & < \sum_{r=1}^{i-1} N_{r+1,i-1} (1-N_r) \frac{B_r - N_{r+1,i} B_i}{B_r} (a_r - v_{1,r}) \\ & \quad + (-1 + \sum_{k=1}^{i-1} (1-N_k) \frac{N_{k+1,i-1}^2 N_i B_i}{B_k}) \sum_{r=1}^{i-1} N_{r+1,i-1} (1-N_r) (a_r - v_{1,r}) \\ & = \sum_{r=1}^{i-1} [-\frac{N_{r+1,i} B_i}{B_r} + \sum_{k=1}^{i-1} (1-N_k) \frac{N_{k+1,i-1}^2 N_i B_i}{B_k}] N_{r+1,i-1} (1-N_r) (a_r - v_{1,r}) \\ & = U_i. \end{aligned}$$

2). We use induction to show that U_j increases as j decreases.

i). We prove $U_i < U_{i-1}$. Separating $a_{i-1} - v_{1,i-1}$ term in U_i provides

$$\begin{aligned} U_i &= \left[-\frac{N_i B_i}{B_{i-1}} + \sum_{k=1}^{i-1} (1-N_k) \frac{N_{k+1,i-1}^2 N_i B_i}{B_k} \right] (1-N_{i-1}) (a_{i-1} - v_{1,i-1}) \\ &\quad + \sum_{r=1}^{i-2} \left[-\frac{N_{r+1,i} B_i}{B_r} + \sum_{k=1}^{i-1} (1-N_k) \frac{N_{k+1,i-1}^2 N_i B_i}{B_k} \right] N_{r+1,i-1} (1-N_r) (a_r - v_{1,r}) \end{aligned}$$

Since $\sum_{k=1}^{i-1} (1-N_k) \frac{N_{k+1,i-1}^2 N_i B_i}{B_k} < \frac{N_i B_i}{B_{i-1}}$ from (A8), the $a_{i-1} - v_{1,i-1}$ term of the above expression is negative. Thus, relaxing it using (A5) gives

$$\begin{aligned} U_i &< \left[-\frac{N_i B_i}{B_{i-1}} + \sum_{k=1}^{i-1} (1-N_k) \frac{N_{k+1,i-1}^2 N_i B_i}{B_k} \right] (1-N_{i-1}) \sum_{r=1}^{i-2} [N_{r+1,i-2} (1-N_r) (a_r - v_{1,r})] \\ &\quad + \sum_{r=1}^{i-2} \left[-\frac{N_{r+1,i} B_i}{B_r} + \sum_{k=1}^{i-1} (1-N_k) \frac{N_{k+1,i-1}^2 N_i B_i}{B_k} \right] N_{r+1,i-1} (1-N_r) (a_r - v_{1,r}) \\ &= \sum_{r=1}^{i-2} \left(\sum_{k=1}^{i-2} \frac{N_{k+1,i-1}^2 N_i B_i (1-N_k)}{B_k} - \frac{N_{r+1,i-2} N_{i-1}^2 N_i B_i}{B_r} \right) N_{r+1,i-2} (1-N_r) (a_r - v_{1,r}) \\ &= U_{i-1} \end{aligned}$$

where the equality comes from collecting and simplifying the $a_r - v_{1,r}$ terms.

ii). We prove $U_j < U_{j-1}$ for $2 \leq j \leq i$. Separating $a_{j-1} - v_{1,j-1}$ term in U_j provides

$$\begin{aligned} U_j &= \left(\sum_{k=1}^{j-1} \frac{N_{k+1,i-1}^2 N_i B_i (1-N_k)}{B_k} - \frac{N_{j,i-1}^2 N_i B_i}{B_{j-1}} \right) (1-N_{j-1}) (a_{j-1} - v_{1,j-1}) \\ &\quad + \sum_{r=1}^{j-2} \left(\sum_{k=1}^{j-1} \frac{N_{k+1,i-1}^2 N_i B_i (1-N_k)}{B_k} - \frac{N_{r+1,j-1} N_{j,i-1}^2 N_i B_i}{B_r} \right) N_{r+1,j-1} (1-N_r) (a_r - v_{1,r}). \end{aligned}$$

Since

$$\begin{aligned} &\sum_{k=1}^{j-1} \frac{N_{k+1,i-1}^2 N_i B_i (1-N_k)}{B_k} - \frac{N_{j,i-1}^2 N_i B_i}{B_{j-1}} \\ &= -\frac{N_i B_i N_{i-1,j}^2 N_{j-1}}{B_{j-1}} + \frac{N_i B_i N_{i-1,j-1}^2 (1-N_{j-2})}{B_{j-2}} + \dots + \frac{N_i B_i N_{i-1,2}^2 (1-N_1)}{B_1} \\ &< N_{i-1,j}^2 \left[-\frac{N_i B_i N_{j-1}}{B_{j-1}} + \frac{N_i B_i N_{j-1}}{B_{j-1}} (1-N_{j-2}) + \dots + \frac{N_i B_i N_{j-1,2}}{B_{j-1}} (1-N_1) \right] \\ &= -N_{i-1,j}^2 \frac{N_i B_i N_{j-1,1}}{B_{j-1}} < 0, \end{aligned}$$

we have

$$\begin{aligned}
U_j &< \left(\sum_{k=1}^{j-1} \frac{N_{k+1,i-1}^2 N_i B_i (1-N_k)}{B_k} - \frac{N_{j,i-1}^2 N_i B_i}{B_{j-1}} \right) (1-N_{j-1}) \sum_{r=1}^{j-2} N_{r+1,j-2} (1-N_r) (a_r - v_{1,r}) \\
&+ \sum_{r=1}^{j-2} \left(\sum_{k=1}^{j-1} \frac{N_{k+1,i-1}^2 N_i B_i (1-N_k)}{B_k} - \frac{N_{r+1,j-1} N_{j,i-1}^2 N_i B_i}{B_r} \right) N_{r+1,j-1} (1-N_r) (a_r - v_{1,r}) \\
&= \sum_{r=1}^{j-2} \left(\sum_{k=1}^{j-2} \frac{N_{k+1,i-1}^2 N_i B_i (1-N_k)}{B_k} - \frac{N_{r+1,j-2} N_{j,i-1}^2 N_i B_i}{B_r} \right) N_{r+1,j-2} (1-N_r) (a_r - v_{1,r}) \\
&= U_{j-1}.
\end{aligned}$$

Now, if we can show $U_2 < 0$, the proof is completed.

$$\begin{aligned}
U_2 &= \left(\frac{N_{2,i-1}^2 N_i B_i (1-N_1)}{B_1} - \frac{N_{2,i-1}^2 N_i B_i}{B_1} \right) (1-N_1) (a_1 - v_1) \\
&= -\frac{N_1 N_{2,i-1}^2 N_i B_i}{B_1} (1-N_1) (a_1 - v_1) < 0
\end{aligned}$$

Proof of Proposition 11

First, from proposition 1, before integration, Q_2 is $N_2 B_2 [a_2 - v_2 - v_1 - (1-N_1)(a_1 - v_1)]$. After integration, it is easy to show that Q_2 is $N_2 B_2 [a_2 - v_2 - v_1]$, and thus, Q_2 always increases with integration. Similarly, p_1 changes from $(1-N_1)a_1 + N_1 v_1$ to $(1-N_1)(a_3 - v_3) + N_1 v_1$, where $a_1 = [N_2 B_2 (a_2 - v_2) + N_3 B_3 (a_3 - v_3)] / (N_2 B_2 + N_3 B_3)$. Therefore, if $a_2 - v_2 > a_3 - v_3$, the resource price p_1 decreases; if $a_2 - v_2 = a_3 - v_3$, p_1 remains unchanged; and if $a_2 - v_2 < a_3 - v_3$, p_1 increases. Q_3 and π_3 increase (decrease) iff p_1 increases (decreases).

As to the profit $\pi_1 + \pi_2$, before the integration,

$$\pi_1 + \pi_2 = (1-N_2)^2 [(B_2 N_2 + B_3 N_3)(a_1 - v_1)^2 + B_2 (a_2 - v_2 - v_1 - (1-N_2)(a_1 - v_1))^2].$$

After the integration, it changes to $\pi_{\text{joint}} = (1-N_2)^2 [B_3 N_3 (a_3 - v_3 - v_1)^2 + B_2 (a_2 - v_2 - v_1)^2]$. Define $(a_2 - v_2 - v_1) / (a_3 - v_3 - v_1)$ as x . After collect terms, we have

$$\begin{aligned}
\Delta &= \pi_{\text{joint}} - (\pi_1 + \pi_2) \\
&= \frac{(1-N_2^2)(a_3 - v_3 - v_1)}{(B_2 N_2 + B_3 N_3)^2} \left\{ B_2 N_2 [B_2 N_2 (1-N_2 - N_2^2) + B_3 N_3 (2-3N_2)] x^2 \right. \\
&\quad \left. + 2B_3 N_3 (B_3 N_3 - B_2 N_2^3 - 2B_3 N_2 N_3) x + B_3 N_3 (B_2 N_2^2 + 3B_3 N_2 N_3 - B_3 N_3 - B_3 N_2^2 N_3) \right\} \\
&= \frac{(1-N_2^2)(a_3 - v_3 - v_1)}{(B_2 N_2 + B_3 N_3)^2} (\tau x^2 + \beta x + \varepsilon).
\end{aligned}$$

where $\tau := B_2 N_2 [B_2 N_2 (1-N_2 - N_2^2) + B_3 N_3 (2-3N_2)]$, $\beta := 2B_3 N_3 (B_3 N_3 - B_2 N_2^3 - 2B_3 N_2 N_3)$, and $\varepsilon := B_3 N_3 (B_2 N_2^2 + 3B_3 N_2 N_3 - B_3 N_3 - B_3 N_2^2 N_3)$.

Note that when $n_2 = 1$ (i.e. $N_2 = 0.5$), we have $\tau > 0$ and $\beta^2 - 4\tau\varepsilon = -B_2B_3N_3 / 4 < 0$. Therefore, for any x , $\Delta > 0$. When $n_2 \geq 2$ (i.e. $N_2 \geq 2/3$), we have $\tau < 0$ and

$$\beta^2 - 4\tau\varepsilon = 4B_3N_3[B_2N_2^2(N_2^2 + N_2 - 1) + B_3N_3(1 - 2N_2)^2] > 0.$$

Therefore Δ could be either positive or negative, depending on the relative value of $(a_2 - v_2 - v_1) / (a_3 - v_3 - v_1)$. Q.E.D.