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Abstract: To analyze effect of privatizing pay-as-you-go public pension on economic and welfare volatilities, we use overlapping generations model where heterogeneous individuals choose their consumptions and labor supplies for responding to aggregate productivity shocks. We theoretically show that the privatization can cause a trade-off between social welfare and economic volatilities. Quantifying the trade-off via simulations, we find that privatizing Social Security improves output stability by 3.8% with reducing volatilities of labor supply and investment by 25.8% and 6.6%, respectively, at the cost of raising instability of social welfare and consumption by 18.8% and 5.3%, respectively, over the business cycle.

Keywords: public pension privatization, economic stability, welfare volatility, responsiveness to TPF shocks

JEL Code: H22, E32, E62

1. INTRODUCTION

Facing a rapid rise in longevity, the pay-as-you-go (PAYG) public pension system in many countries becomes fiscally nonviable. For example, among the OECD countries, the average public pension debt is projected to reach 124.2% of GDP by 2030 (Disney, 2000). Moreover, in the United States, funds for the Social Security retirement program will be depleted by 2033. To address the need to reform unsustainable PAYG public pension system, various measures have been proposed, among which privatizing public pension draws most of attentions and is actually implemented in some countries. While it naturally eliminates the fiscal insolvency problem of PAYG public pension system, privatization of PAYG public pension inevitably exposes more retirement wealth of individuals to aggregate shocks, entailing changes in individuals' responses to the shocks. As recently observed in the Great Recession, macroeconomic shocks are not easily ignorable. Nevertheless, little is studied on

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how public pension privatization affects economic fluctuation with aggregate shocks. Since public pension funds are quite sizable (e.g., 17.9% of GDP in the US)[1], the impact of public pension privatization would be neither trivial nor limited to a small part of economy. This paper aims to investigate the effect of privatizing PAYG public pension on the volatilities of economy and social welfare in responding to aggregate productivity shocks over the business cycle.

To assess the economic and welfare effects of privatizing PAYG public pension, most of previous studies have examined only steady-state equilibria before and after the privatization (e.g., Hubbard and Judd 1987; Breyer and Straub 1993; İmrohoroğlu et al. 2003; Nishiyama and Smetters 2007; Fuster et al. 2007).[2] However, such steady-state analyses leave us ignorant of how privatizing PAYG public pension affects macroeconomic and welfare responses to aggregate shocks that entail deviations from the steady state. Bovenberg and Uhlig (2008) pointed out that a public pension optimal at steady state is not necessarily optimal *ex-post* when a macroeconomic shock is materialized. Krueger and Kubler (2006) and Olovsson (2010) reported social welfare changes brought by only four different levels of aggregate shocks before and after public pension privatization; nevertheless, they did not analyze effect of the privatization on volatility of economy or social welfare over the business cycle. Moreover, Krueger and Kubler (2006) and Olovsson (2010) did not allow individuals to choose their own labor supply, which limits the applicability of their works since public pension privatization makes a difference in labor supply decisions.

In this paper, we utilize an overlapping generations model, where heterogeneous individuals choose their own consumption and labor supply bearing idiosyncratic risks on

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[1] This is also similar to the average for the countries in the Organization for Economic Co-operation and Development: 19.6% of GDP (OECD 2013).

[2] Hubbard and Judd (1987), Krueger and Kubler (2006), and İmrohoroğlu et al. (2003) regarded an economy without public pension (fully privatized pension) as pre-reform initial state and examined stationary equilibrium after introducing PAYG public pension system. In contrast, Nishiyama and Smetters (2007), and Fuster et al. (2007) treated economy with PAYG public pension system as initial state. Either way, these studies agree in the finding that privatizing PAYG public pension increases steady-state capital and labor supply (and thus output), although they disagree in overall welfare outcome of the privatization.

earning ability and longevity, to obtain general equilibria of pre- and post- privatization economies. We theoretically prove that when privatizing PAYG public pension raises steady-state aggregate labor supply and capital with reducing level deviation of total labor supply from its steady state for responding to aggregate productivity shocks, the privatization improves stabilities of total output, labor supply, and investment, at the cost of increasing instabilities (volatilities) of aggregate consumption and social welfare over the business cycle.

To quantify our theoretical findings, we calibrate our model to match US economy data and obtain stationary equilibria before and after privatization of Social Security PAYG public pension. We first find that the privatization raises steady-state levels of total labor supply, capital, and output, which corroborates findings of the previous studies (e.g., Hubbard and Judd 1987; Breyer and Straub 1993; İmrohoroğlu et al. 2003; Nishiyama and Smetters 2007; Fuster et al. 2007). By introducing the same series of 1000 total productivity factor (TPF) shocks to the two steady-state economies that are identical except for the Social Security public pension, the privatization is found to reduce variation in total labor supply for responding to the aggregate shocks. Above all, we find that the public pension privatization enhances economic stability, as it reduces instability of total output by 3.8% with reducing volatilities of aggregate labor supply and investment by 25.8% and 6.6%, respectively. However, this benefit of improvement in economic stability comes at cost: The privatization also raises instability of social welfare by 18.8% with increasing volatility of aggregate consumption by 5.3% over the business cycle.

The trade-off between social welfare and economic volatility stems from the exposure of retirement wealth to macroeconomic shocks. Without PAYG public pension, individuals face more loss (gain) in their retirement wealth from a negative (positive) aggregate shock, so they decrease (increase) their labor supplies and savings by less margin than before public pension privatization, while they reduce (raise) their consumptions more. As a result of such effort for securing resource for retirement consumption, the privatization decreases volatilities of aggregate labor supply and investment, which in turn entails decrease in total output instability over the business cycle. However, such a beneficial effect of the privatization — improvement in economic stability — turns out not free of side effect. That is, volatility of social welfare is increased by the privatization, since individuals end up with suffering (enjoying) more by more (less) work and less (more) consumption facing recession (expansion) than before the privatization.

In short, this paper makes contributions by theoretically showing that privatizing PAYG public pension can improve economic stability (decrease volatilities of total output, labor supply and investment) at the cost of increasing volatilities of social welfare and aggregate consumption as well as by empirically estimating such effects with data of US economy.

The rest of this paper unfolds as follows. Section 2 presents our overlapping generations model from which we derive theoretical property regarding impact on economic and welfare volatilities of privatizing PAYG public pension. For empirical analysis, Section 3 calibrates our model to the US economy, by which we simulate aggregate productivity shocks. Section 4 compares the simulation results to estimate the effect of the privatization on economic and welfare volatilities. Section 5 concludes the paper.

2. MODEL

Consider an economy that is inhabited by heterogeneous individuals who are different in age i and earning ability e. Each period t, individuals are born with zero endowment and live no longer than I (i.e., $i \in \{1, \cdots, I\}$). Thus, at any given time, I different age cohorts co-exist in the economy. With probability m_i, living age-i individuals survive to be of age $i+1$, while the total population grows at a constant rate n.[3] Moreover, for each period, individuals' earning ability $e \in E = [\underline{e}, \overline{e}] > 0$ is subject to idiosyncratic shocks.

[3] By using two sets of demographic parameters (total population growth rate and mortality rate at each age) instead of one, the population ratio of different age cohorts can be time-invariant in overlapping generations of the finitely lived agents.

Each period t, given the state of the economy and government policies, individuals decide their consumptions, labor supplies, and savings (investments) to maximize expected utility for their remaining lifetimes, which is the sum of their current and future within-period utilities at the present value. The within-period utility function is $u(c_{t,i}, l_{t,i})$ where $c_{t,i}$ and $l_{t,i}$ are consumption and labor supply, respectively, of an age-i individual for period t. As usual, $u(c_{t,i}, l_{t,i})$ is increasing in consumption (i.e., $u_c > 0$) and is decreasing in labor supply (i.e., $u_l < 0$). The amount of time given to every individual is one for each period; thus, $l_{t,i} \in [0,1]$ for $\forall t$ and $\forall i$. When making their decisions, each individual faces intertemporal budget constraints as follows: for $\forall t$ and $\forall i$,

$$w_t e l_{t,i}(1-\tau_p-\tau_l)+(1+(1-\tau_k)r_t)k_{t,i}+q_t+1_R(i)a_{t,i} \geq c_{t,i}+(1+g)k_{t+1,i} \text{ and } k_{t,i} \geq 0\,, \quad (1)$$

where w_t and r_t are the market wage rate and interest rate, respectively, of period t; τ_p is the payroll tax rate for the public pension whereas τ_l and τ_k are rates of taxes on labor income and capital gain, respectively; q_t is accidental bequest from those who die at the end of period t. Moreover, $1_R(i)$ is a binary indicator for whether an age-i individual is eligible to receive public pension benefit (taking the value of one if he is eligible, zero otherwise); and the amount of potential public pension benefit is $a_{t,i}$, whose details are specified by public pension policies. While $k_{t,i}$ is the investment (savings) made in the previous period, $k_{t+1,i}$ is chosen as savings in the current period t. Individuals face a borrowing constraint ($k_{t,i} \geq 0$). Furthermore, insurances for earning ability and mortality are not provided in the private market.[4]

[4] As a result, annuity is not supplied in the private market.

For simplicity, as in previous studies like Nishiyama and Smetters (2007) and Fuster et al. (2007), bequest left accidentally by individuals who do not survive to the next period is not gone with the deceased but equally distributed to surviving individuals. That is,

$$q_t = \sum_{i=1}^{I}(1-m_i)\int_{E\times K\times A} k_{t+1,i}(\mathbf{s}_t)dP_t(\mathbf{s}_t)\{\sum_{i=1}^{I} m_i \int_{E\times K\times A} dP_t(\mathbf{s}_t)\}^{-1}, \quad (2)$$

where $\mathbf{s}_t = (i, e, k_{t,i}, a_{t,i})$ is a vector of state variables at the beginning of period t. The distribution of the state variables $P_t(\mathbf{s}_t)$ evolves as follows:

$$p_{t+1}(i+1, e, k', a') = \frac{m_i}{1+n}\int_{E\times K\times A} 1(k' = k_{t+1,i}(\mathbf{s}_t) + q_t) \times 1(a' = a_{t+1,i+1}(w_t e l_{t,i}(\mathbf{s}_t), a_{t,i}))dP_t(\mathbf{s}_t), \quad (3)$$

where $p_{t+1}(\mathbf{s}_{t+1})$ refers to the probability measure of the state variables $\mathbf{s}_{t+1}$; sets of E, K, and A denote the supports for earning ability, wealth (capital), and potential amount of public pension benefit, respectively; $1(\cdot)$ is an indicator function that takes the value of one if the statement inside the parenthesis is true and zero otherwise. Moreover, $k_{t+1,i}(\mathbf{s}_t)$ and $l_{t,i}(\mathbf{s}_t)$ are decision rules (policy functions) of savings and labor supply, respectively, obtained from age-i individuals' maximizing their expected utility for their remaining lifetimes in period t.

In short, the decision problem that an age-i individual solves for period t is written as

$$V_i(\mathbf{s}_t) = \max_{\{c_{t,i}, l_{t,i}, k_{t+1,i}\}} u(c_{t,i}, l_{t,i}) + m_i \beta E_t[V_{i+1}(\mathbf{s}_{t+1})] \qquad (4)$$

where β is the time preference parameter and $V_i(\cdot)$ is the maximized utility function (value function) for the remaining lifetime of an age-i individual subject to the budget constraint (1) and government policies.

In addition, there exists a representative firm in the economy which produces output Y_t for period t, following a Cobb-Douglas technology as below.

$$Y_t = F(K_t, L_t) = z_t K_t^{\alpha} L_t^{1-\alpha}, \quad (5)$$

where K_t is aggregate capital input invested by individuals in the economy and L_t is the total sum of individuals' labor supplies in efficiency units. In particular, z_t is TPF that is subject to a random shock as

$$z_{t+1} = z_t^{\rho} \exp(\varepsilon_{t+1}) \text{ where } \varepsilon_t \sim WN(0, \sigma_z^2) \text{ for } \forall t. \quad (6)$$

The firm solves its profit maximization problem of $\max_{\{K_t, L_t\}} z_t K_t^{\alpha} L_t^{1-\alpha} - (r_t + \delta) K_t - w_t L_t$, which begets the following decision rules:

$$F_K(K_t, L_t) = r_t + \delta \quad (7)$$

$$F_L(K_t, L_t) = w_t \quad (8)$$

where δ is a depreciation rate of capital.

The government chooses labor income tax rate τ_l to balance its budget as follows.

$$G + \sum_{i=R}^{I} \int_{E \times K \times A} a_{t,i}(\mathbf{s}_t) dP_t(\mathbf{s}_t) = \sum_{i=1}^{I} \int_{E \times K \times A} \{ w_t e l_{t,i}(\mathbf{s}_t)[\tau_l + \tau_p] + \tau_k r_t k_{t,i}(\mathbf{s}_t) \} dP_t(\mathbf{s}_t), \quad (9)$$

where G is a given required government expenditure and R is the "normal retirement age," which is the legal age from which an individual is eligible to receive fully vested pension benefit $a_{t,i}$. Following the previous studies, individuals cannot receive the public pension benefit $a_{t,i}$ before R, whereas individuals cannot delay or stop receiving the benefit after R. That is,

$$1_R(i) = \begin{cases} 0 \text{ if } i < R \\ 1 \text{ if } i \geq R \end{cases}. \quad (10)$$

Notice, however, that our model allows individuals to choose to keep working even after R.

Taking all the three parties (individuals, firm, and government) together, this economy as a whole meets the following resource constraint

$$Y_t = C_t + (1+n)(1+g)K_{t+1} - (1-\delta)K_t + G + \sum_{i=R}^{I} \int_{E \times K \times A} a_{t,i}(\mathbf{s}_t) dP_t(\mathbf{s}_t), \quad (11)$$

where $C_t \equiv \sum_{i=1}^{I} \int_{E\times K\times A} c_{t,i}(\mathbf{s}_t) dP_t(\mathbf{s}_t)$ is aggregate consumption. Via competitive markets where each party solves their own maximization problem, this economy reaches a stationary recursive general equilibrium that is defined as follows.

Given government policies $\{G, R, \tau_p, \tau_k\}$ and the public pension benefit formula, the stationary recursive competitive equilibrium of this economy is a set of value functions $\{V_i(\mathbf{s}_t)\}_{i=1}^{I}$; decision rules $\{c_{t,i}(\mathbf{s}_t), l_{t,i}(\mathbf{s}_t), k_{t+1,i}(\mathbf{s}_t), a_{t,i}(\mathbf{s}_t)\}_{i=1}^{I}$; the associated distribution of the state variables defined by its probability measure $p_t(\mathbf{s}_t)$ following (3); a lump-sum transfer of accidental bequest q_t; labor income tax rate τ_l; and the prices of labor w_t and capital r_t that satisfy the following conditions for $\forall t$:

(i) Given the government policies, factor prices, and transfer of accidental bequest, all individuals' decision rules solve their problem of (4).

(ii) The representative firm maximizes its profit by satisfying (7) and (8) with the factor markets cleared as below:

$$K_t = \sum_{i=1}^{I} \int_{E\times K\times A} k_{t,i}(\mathbf{s}_t) dP_t(\mathbf{s}_t) \quad (12)$$

$$L_t = \sum_{i=1}^{I} \int_{E\times K\times A} e l_{t,i}(\mathbf{s}_t) dP_t(\mathbf{s}_t), \quad (13)$$

which satisfy (11) as well.

(iii) The government sets τ_l by (9) and distributes q_t defined by (2).

(iv) This economy reaches a steady state if

$$p_t(\mathbf{s}) = p_{t+1}(\mathbf{s}) \text{ for } \forall \mathbf{s} \in \{1, \cdots, I\} \times E \times K \times A \text{ and } \forall t. \quad (14)$$

In a steady state, therefore, all the variables drop their time subscript.

Having obtained steady-state general equilibrium, we further allow this economy to fluctuate, instead of staying steadfastly at the steady state, by letting it respond to macroeconomic shocks. In our model, such an aggregate shock is described as an unexpected change in TPF following (6) and propagates throughout the entire economy via reactions of individuals and the firm. According to Uhlig (1999) that provides standard tools for real business cycle analyses, we express the equilibrium laws of such reactions in terms of log deviations from the steady state by approximating them in the form of linear function. From intra- and inter-temporal conditions for the individuals' dynamic optimization of (4) with respect to labor supply, consumption, and savings, we derive log-linearized optimal reactions as follows:

$$\frac{1}{\eta} e\hat{l}_{t,i} = \hat{Y}_t - \frac{1}{\bar{L}} \Big(\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i} \bar{l}_i(\mathbf{s}) dP(\mathbf{s}) \Big) - \hat{c}_{t,i} \quad \text{for} \quad \forall i \in \{1, \cdots, I\} \qquad (15)$$

$$\hat{c}_{t,i} = E[\hat{Y}_{t+1} - \frac{1}{\bar{K}} \sum_{i=1}^{I-1} \int_{E\times K\times A} \hat{k}_{t+1,i+1} \bar{k}_{i+1}(\mathbf{s}) dP(\mathbf{s}) - \hat{c}_{t+1,i+1}] \quad \text{for} \quad \forall i \in \{1, \cdots, I-1\} \qquad (16)$$

$$(\hat{Y}_t - \frac{1}{\bar{L}} \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i} \bar{l}_i(\mathbf{s}) dP(\mathbf{s}) + \hat{l}_{t,i}) \bar{w} e\bar{l}_i (1 - \tau_l - \tau_p) + (1 - \tau_k) \bar{r} \bar{k}_i (\hat{Y}_t - \frac{1}{\bar{K}} \sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i} \bar{k}_i(\mathbf{s}) dP(\mathbf{s})) + (1 + (1 - \tau_k) \bar{r}) \bar{k}_i \hat{k}_{t,i} = \bar{c}_i \hat{c}_{t,i} + (1+g) \bar{k}_{i+1} \hat{k}_{t+1,i+1} \quad \text{for} \quad \forall i \in \{1, \cdots, I\} \qquad (17)$$

where $\bar{x}$ is a steady-state value of variable x_t and $\hat{x}_t = \log(\frac{x_t}{\bar{x}})$. As Uhlig (1999) shows, when the value of x_t lies in the neighborhood of its steady-state value of $\bar{x}$, $\hat{x}_t$ captures volatility (responsiveness) of variable x_t since $100\hat{x}_t$ approximates % deviation of variable x_t from its steady-state value $\bar{x}$ in period *t*. In this line, variation of variable x_t (level-deviation from its steady-state value) in period *t* is measured as $\hat{x}\bar{x}$. From the representative firm's production (5) and (6), we also obtain the following equilibrium laws of motion through which a TPF shock can spread throughout the whole economy.

$$\hat{Y}_t = \hat{z}_t + \frac{\alpha}{\bar{K}} \sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i} \bar{k}_i(\mathbf{s}) dP(\mathbf{s}) + \frac{1-\alpha}{\bar{L}} \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i} \bar{l}_i(\mathbf{s}) dP(\mathbf{s}) \qquad (18)$$

$$\hat{z}_t = \rho \hat{z}_{t-1} + \varepsilon_t. \qquad (19)$$

After all, by introducing a TPF shock with an unexpected change in ε_t of (19), which is initially zero at the steady-state, and then by solving a system of the above linear equations (15), (16), (17), (18), and (19), we obtain economic and welfare responses to the exogenous macroeconomic shock.

From our general model, we can derive a theoretical statement on effect of public pension privatization on economic and social welfare volatilities as follows.

Proposition 1. When privatization of PAYG public pension decreases variation in aggregate labor supply responding to TPF shocks with raising steady-state levels of aggregate labor supply and capital, the privatization decreases volatility of total output over the business cycle by reducing volatilities of aggregate labor supply and investment.

Proof. See Appendix.

As various previous studies (e.g., Hubbard and Judd 1987; Breyer and Straub 1993; İmrohoroğlu et al. 2003; Nishiyama and Smetters 2007; Fuster et al. 2007) have shown, privatization of PAYG public pension raises steady-state aggregate labor supply and capital, since the privatization removes distortion brought by tax for public pension imposed on labor earnings, while it increases steady-state total capital which was crowded out by the PAYG public pension. In place of PAYG public pension, individuals save more to secure resource for their retirement consumption than before the privatization.

Intuitively, as individuals experience more loss (gain) of retirement wealth in recession (expansion) after privatization of PAYG public pension, they reduce (increase) their labor supplies and savings by less margin than before the privatization. As a result of such wealth effect from increased exposure of retirement wealth to aggregate shocks, the privatization decreases volatilities of aggregate labor supply and investment. As volatility is measured relative to the steady state level, given that steady-state labor supply and capital is increased by the privatization, such decreases in volatilities of labor supply and investment reduces output volatility only if their variation (level of deviation) is large enough.

Clearly, improvement of economy stability from decreases in volatilities of total output, labor supply, and investment is a desired feature of PAYG public pension privatization. Nonetheless, the privatization can simultaneously raise instability of social welfare over the business cycle. As usual, social welfare is defined as weighted sum of utilities of heterogeneous individuals with their population weights.

Proposition 2. When privatization of PAYG public pension decreases variation in aggregate labor supply responding to TPF shocks with raising steady-state levels of aggregate labor supply and capital, the privatization increases volatility of social welfare and total consumption over the business cycle while decreasing volatility of total output.

Proof. See Appendix.

Imposing more risk on the resources for post-retirement consumption after public pension privatization increases volatility of aggregate consumption, as individuals reduce (increase) consumption more facing more loss (gain) of retirement wealth in recession (expansion) than before the privatization. Therefore, in recession (expansion), individuals suffer (enjoy) more by more (less) labor and less (more) consumption in the after-privatization economy than in the before-privatization economy, entailing the rise in instability of social welfare.

To quantify our theoretical findings of the trade-off between economic stability and social welfare volatility, in the next section, we calibrate our model to the US economy under the current PAYG public pension to estimate economic and welfare volatilities before and after privatizing the public pension by introducing 1000 macroeconomic shocks on productivity.

3. CALIBRATION

We calibrate our overlapping generations model to match the United States economy under the current Social Security retirement program, as our baseline (pre-reform) economy. One period in our model is equivalent to one year. Individuals of age 1 ($i=1$) in our model correspond to 21-year-old individuals. The sequence of survival rates (m_i) is obtained from the data of life tables released by the United States Centers for Disease Control and Prevention, based on which we set $I = 80$. For the within-period utility function of each individual, we use

$$u(c_{t,i}, l_{t,i}) = \log(c_{t,i}) - \psi \frac{l_{t,i}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}. \qquad (20)$$

As you may notice, the coefficient of relative risk aversion is set to 1, following Chetty (2006) that estimates the risk aversion parameter using data on observed labor supply behavior. This is lower than the values used in the previous studies such as 4 in Fuster et. al (2007) or 2 in Nishiyama and Smetters (2007) which are chosen without considering consistency with labor supply or with any other empirical studies. On the other hand, for the Frisch elasticity of labor supply which is η, we select the value of 1.5 because Chang and Kim (2014) proved this value to match real-data volatility of labor hours.[5] Fiorito and Zanella (2012) found that estimates for Frisch elasticity consistent with the observed volatility in aggregate labor supply range from 1.1 to 1.7. Moreover, the model that Chang and Kim (2014) used is closer to our model than any others like Cho and Cooley (1994) or Imai and Keane (2004).[6] Although no clear consensus on the value of Frisch elasticity is yet established, İmrohoroğlu and Kitao (2009) showed that steady-state outcome of public pension privatization is not sensitive to the values of Frisch elasticity.[7]

Regarding other preference parameters for our simulation analyses, the value of β is calibrated to generate the interest rate of 3.48% in steady state under the current Social

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[5] Instead of 1.5, Fuster et. al (2007) adopt 1 as the Frisch elasticity following Chang and Kim (2006). However, Chang and Kim (2006) admitted that their estimate, 1, does not generate enough fluctuation in working hours as real data.

[6] It also is very close to the estimate of Browning, Hansen and Heckman (1999) which is 1.6.

[7] Early estimates such as Altonji (1986) are between 0 and 0.5, which are not consistent with the observed volatility of aggregate labor supply over the business cycle (Chetty et al. 2011). Imai and Keane (2004) extended standard model by incorporating unobservable human capital accumulation and estimated the Frisch elasticity to be 3.85 which is greater than estimates of any other studies such as Cho and Cooley (1994), Chang and Kim (2006), and Chang and Kim (2014).

Security public pension system, while the value of ψ (parameter that captures disutility of working) is selected to beget the associated steady-state employment rate as 64%. Basically, values of all the parameters, including survival rates (m_i), are chosen to match the values of recent aggregate US data which are averaged from year 2000 to year 2010. In particular, data on the capital share of output are taken from the US Bureau of Economic Analysis; the capital depreciation rate is based on the estimates of Alice Albonico, Sarantis Kalyvitis, and Evi Pappa (2014); and data on the rates of population growth and economic growth and on the government spending in terms of share of GDP are procured from the World Bank database. According to these data, we calibrate the government expenditure (G) to comprise 15.4% of GDP. Moreover, for ρ and σ_z of the production technology proceeding in (6), we adopt the estimates of Komunjer and Ng (2011). The values of parameters chosen for our simulation analyses are displayed in **Table 1**.

Table 1] Parameters of the Baseline Economy

Capital share of output	α	0.317
Depreciation rate of capital	δ	0.117
Rate of output growth	g	0.019
Population growth rate	n	0.009
Total productivity factor	z	1
Autocorrelation of total productivity shock	ρ	0.9
Standard deviation of total productivity factor	σ_z	0.9
Social Security payroll tax rate	τ_p	0.124
Capital gain tax rate	τ_k	0.25
Time preference (discount factor)	β	0.996
Weight on disutility from work	ψ	5.160
Frisch elasticity of labor supply	η	1.5

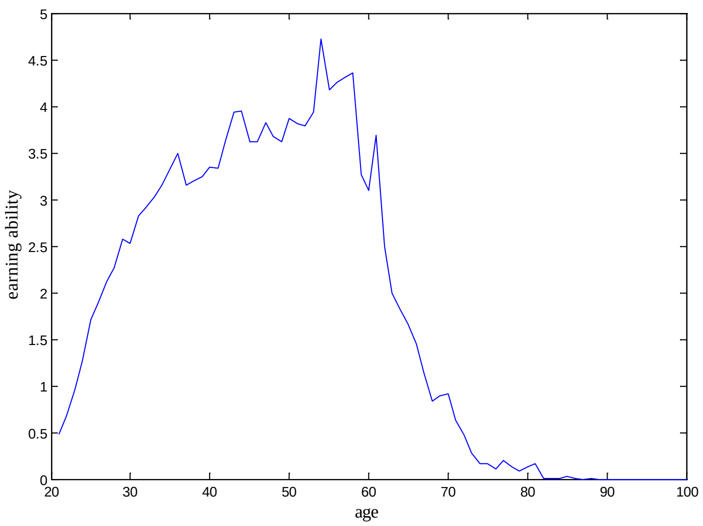
For the before-privatization baseline economy, τ_p is set at 12.4% combining the Social Security payroll tax rate of 6.2% paid by employees and by their employers, respectively. Following the Social Security benefit formula, pension benefit $a_{t,i}$ is calculated as follows.[8]

[8] For further details, visit http://www.ssa.gov/oact/cola/piaformula.html.

$$a_{t+1,i+1} = \begin{cases} 0.9\bar{a}_{t,i} \text{ if } \bar{a}_{t,i} \le 0.22\bar{\mathrm{A}} \\ 0.9(0.02\bar{\mathrm{A}})+0.32(\bar{a}_{t,i} - 0.02\bar{\mathrm{A}}) \text{ if } 0.22\bar{\mathrm{A}} < \bar{a}_{t,i} \le 1.33\bar{\mathrm{A}} \\ 0.9(0.02\bar{\mathrm{A}})+0.32(0.11\bar{\mathrm{A}} - 0.02\bar{\mathrm{A}})+0.15(\bar{a}_{t,i} - 0.11\bar{\mathrm{A}}) \text{ if } 1.33\bar{\mathrm{A}} < \bar{a}_{t,i} \le 2.64\bar{\mathrm{A}} \\ 0.9(0.02\bar{\mathrm{A}})+0.32(0.11\bar{\mathrm{A}} - 0.02\bar{\mathrm{A}})+0.15(2.47\bar{\mathrm{A}} - 0.11\bar{\mathrm{A}}) \text{ if } 2.64\bar{\mathrm{A}} < \bar{a}_{t,i} \end{cases} \quad (21)$$

if $i < R$, otherwise $a_{t+1,i+1} = a_{t,i}$, where $\bar{\mathrm{A}} \equiv \sum_{i=1}^{I} \int_{E\times K\times A} w_t el_{t,i}(\mathbf{s}_t) dP_t(\mathbf{s}_t) \{\sum_{i=1}^{I} \int_{E\times K\times A} dP_t(\mathbf{s}_t)\}^{-1}$ (average labor income of the economy) and $\bar{a}_{t,i} \equiv \frac{1}{i}\sum_{h=1}^{i} w_{t-i+h} el_{t-i+h,h}$ (average of the past labor incomes of an age-i individual in period t). Individuals become eligible to receive the pension benefit at the normal retirement age of 65, so we set R at 45.

Figure 1] Earning Abilities across Different Ages



As previous studies like Nishiyama and Smetters (2007) did, earning ability e is approximated with data of individuals' annual earnings from the Panel Study of Income Dynamics (PSID) by averaging over the three waves (years of 2003, 2005, and 2007).[9] As described in Section 2, individuals of the same age can have different earning abilities, as the earning ability of each individual is subject to idiosyncratic shocks each year. To reflect the ensuing heterogeneity in earning abilities, we first divide each age cohort of the PSID into four income groups with thresholds of $20,000, $40,000, and $60,000; and then we average

9 Data from those who reported hourly/weekly earnings are converted into annual income based on hours worked.

annual earnings of each of the four income groups for each age cohort. The resulting 4 by 80 matrix is used as earning abilities in our simulation. In addition, to capture the idiosyncratic uncertainty on earning ability of any given age, the probability for an individual to have one of the four levels of earning ability is approximated with the population share of the corresponding income group in the age cohort of the PSID. In the end, we obtain the profile of age and expected earning ability, which is plotted in **Figure 1**.[10]

4. SIMULATION RESULTS

4.1. Steady State Economies

We obtain steady-state equilibria of the pre-privatization and post-privatization economies by solving our model by meeting the conditions from (1) to (14) with the within-period utility function of (20) and the parameters presented in the previous section. The post-privatization economy is different from the pre-privatization baseline economy only in public pension policy by setting $\tau_p = 0$ and $a_{t,i} = 0$ for $\forall t$ and $\forall i$ for the former. The approach of comparing steady-state equilibria of the economy before the privatization starts and the economy after the privatization is completed prevents our results from hinging upon numerous arbitrary assumptions on the transition process such as how fast the privatization is phased-in; how to finance and distribute necessary compensations for the existing retirees and workers who already have paid payroll taxes, and the like.[11]

As shown in **Table 2** that summarizes steady-state equilibria before and after the privatization, privatizing PAYG Social Security public pension raises steady-state aggregate capital by 9.3%, which leads the interest rate to fall to 2.76% from 3.48%. Individuals save more for maintaining their post-retirement consumption, as the government ceases to provide

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[10] The hump shape in **Figure 1** is very similar to the shape of estimated earning abilities over the lifetime by Hansen (1993). Unlike his work, however, our distribution covers more age groups (older than 65).

[11] Neither in theory nor in practice, there is no clear consensus on the path of the privatization. However, as Huang et al. (1997) and Kotlikoff et al. (1999) showed, overall welfare consequence of public pension privatization is sensitive to welfare cost from the transition process. In fact, the overall welfare gain (or loss) of privatizing PAYG public pensions differs by the transition path. For instance, Nishiyama and Smetters (2007) and Fuster et al. (2007) found that when compensations for preserving utilities during the privatization process are financed by labor income taxes, public pension privatization ends up with a social welfare loss (despite increased labor supply, capital stock, and output), whereas when the compensations are financed by consumption taxes, public pension privatization generates a social welfare gain.

public pension benefits for them. At the same time, as payroll taxes on labor incomes are no longer collected to finance the Social Security retirement benefits, the privatization removes the labor supply disincentives that the PAYG public pension system imposes. Thus, steady-state aggregate labor supply increases by 1.8% while market wage increases by 2.3%. As a result, the privatization raises steady-state total output by 4.2%. In fact, such a pro-growth effect of public pension privatization resonates with other studies (e.g., Hubbard and Judd 1987; İmrohoroğlu et al. 2003; Nishiyama and Smetters 2007; Fuster et al. 2007).[12]

Table 2] Steady State Economies Before and After Public Pension Privatization

Pension system	$\bar{K}$	$\bar{L}$	$\bar{Y}$	$\bar{C}$	$\frac{\bar{C}}{\bar{Y}}$	$\tau_p + \tau_l$	$\bar{r}$	$\bar{w}$	Social welfare
PAYG	4.159 (100.0)	1.415 (100.0)	1.991 (100.0)	1.069 (100.0)	0.537 (100.0)	0.208 (100)	0.0348 (100.0)	0.961 (100.0)	−0.258 (100.0)
Fully privatized	4.546 (109.3)	1.441 (101.8)	2.074 (104.2)	1.095 (102.4)	0.528 (98.3)	0.203 (97.6)	0.0276 (79.4)	0.983 (102.3)	−0.225 (129.3)

Note: The first row reports the stationary equilibrium of the economy under the current Social Security PAYG public pension system, whereas the second row does so for the economy after completion of privatizing the public pension. Numbers in parentheses refer to rescaled values of variables with the before-privatization steady state as 100 in order to express effects of the public pension privatization in percentage terms.

After the privatization, steady-state total consumption also increases by 2.4%. However, as shown in **Table 2**, the share of aggregate consumption in total output ($\frac{\bar{C}}{\bar{Y}}$) slightly declines, which indicates an additional increase in risk-averse individuals' private savings for their own post-retirement income insurance due to the demise of cross-generation risk sharing that is uniquely provided by the PAYG public pension. After all, as shown in the last column of **Table 2**, the level of social welfare at the steady state reached after completing the public pension privatization is greater than that of our baseline pre-privatization economy.

In reality, economy does not constantly stay at its steady state but fluctuates around the steady state. So, with the *ex-ante* basis of the steady states elaborated in the current section, in the following section, we explore effects of public pension privatization on *ex-post* changes

[12] The margin of the increase in steady-state aggregate capital is lower than that of previous studies because of our coefficient of relative risk aversion is lower than theirs.

in the economy and welfare facing total productivity shocks.

4.2. Economic and Welfare Responses to a TPF Shock

This section illustrates impulse responses to a supply-side macroeconomic shock before and after the privatization. To this end, we introduce an unexpected drop in TPF by deviating the value of ε_t from zero (at the steady state) in (19), and then obtain individuals' impulse response functions from solving (15), (16), (17), (18), and (19), which are aggregated with the population weight. For the sake of relevancy, we select the unexpected fall in ε_t as -1.85, instead of -1, to produce -2.2% point deviation from the steady-state output growth, as it can match not only US GDP growth data of year 2008 but also that of year 2000 when the sign of the shock is reversed.[13] We obtain symmetric results when we take $\varepsilon_t = 1.85$; thus, for efficient presentation, we report only responses of the pre- and post-privatization economies to the negative shock on TPF.

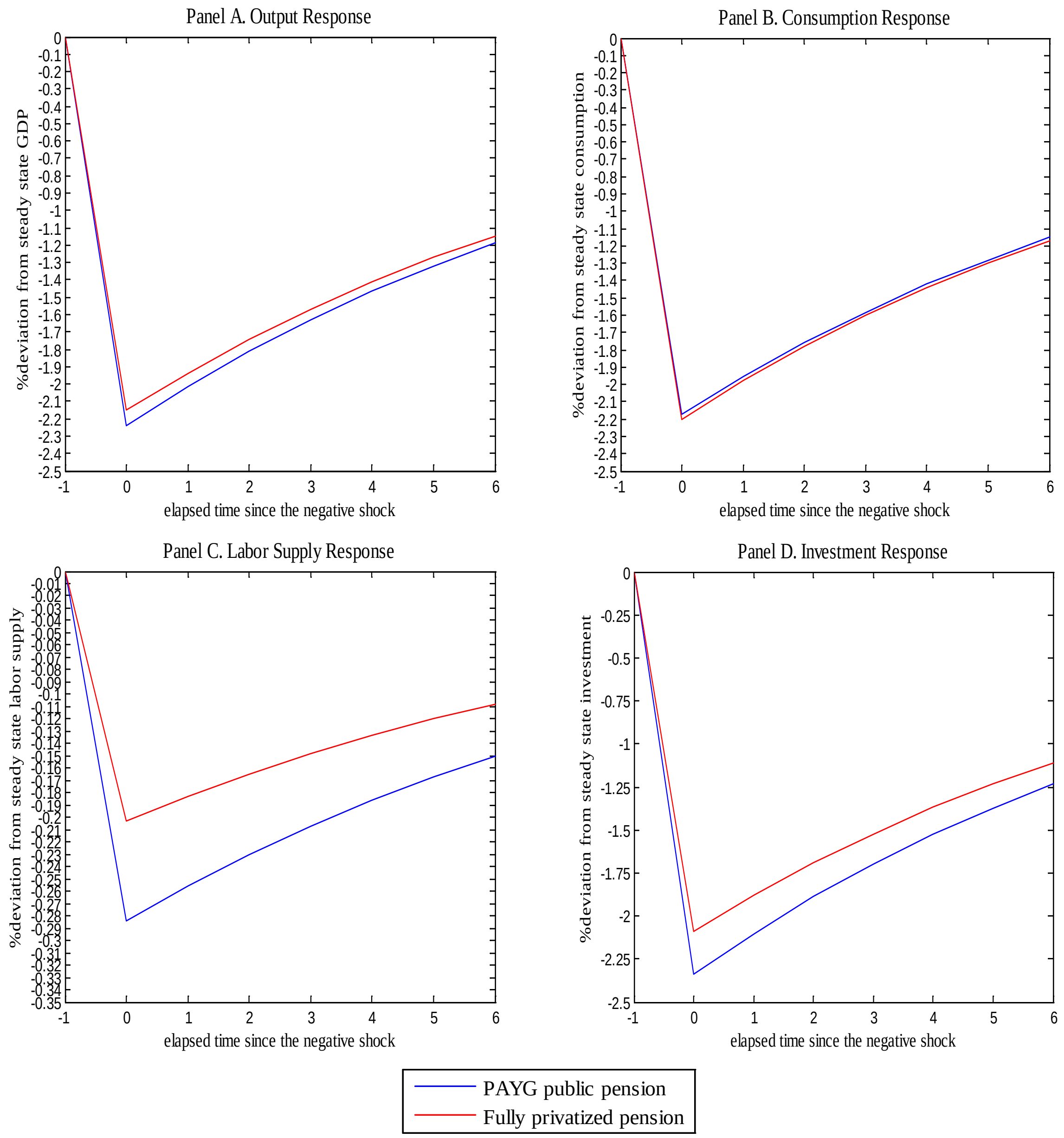
As shown in **Figure 2**, the privatization of PAYG Social Security public pension reduces *ex-post* responses of total output, labor supply, and investment to the TPF shock. In terms of the largest deviation from the steady state which occurs at the moment when the shock hits, the response of total labor supply decreases by 28.6% (from −0.28% point deviation in the pre-privatization economy to −0.20% point deviation in the post-privatization economy); that of aggregate investment declines by 10.7% (from −2.34% points to −2.09% points); and that of total output decreases by 4% (from −2.24% points to −2.15% points). In contrast, the largest response of total consumption increases by 1.4% (from −2.17% points to −2.20% points) due to privatizing the PAYG public pension.

Responding to the negative aggregate productivity shock, individuals reduce their labor supplies and savings in the pre-privatization and post-privatization economies alike. However,

[13] Deviating 2.2% points negatively from the steady-state growth rate (1.9%) returns the growth rate of year 2008 (−0.3%) whereas deviating 2.2% points positively with $\varepsilon_t = 1.85$ matches growth rate of 2000 (4.1%). Moreover, we also conducted the same simulation exercise for the case of $\varepsilon_t = -1$ and obtained the qualitatively same result as the case of $\varepsilon_t = -1.85$.

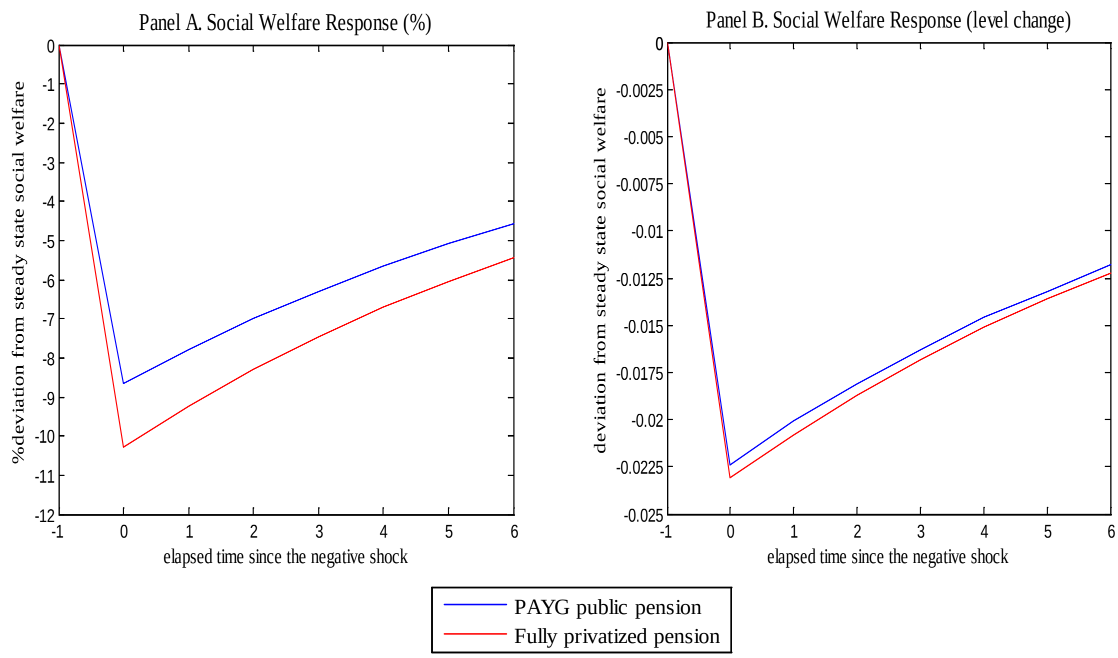
individuals' retirement wealth declines more in the latter, as it is exposed more to the negative shock. As a result, individuals reduce their labor supply and savings to a *lesser* degree (i.e., work more and save more) with decreasing consumption more in the post-privatization economy. In turn, more work and more savings entail less reduction in output of the after-privatization economy than the before-privatization economy.

Figure 2] Impulse Responses of Macroeconomic Variables to a Negative TPF Shock: Before and After Public Pension Privatization



As a consequence of more reduction in consumption and less reduction in labor supply (panel B and panel C in **Figure 2**), facing the negative TPF shock, social welfare declines more in the post-privatization economy. As shown in panel A of **Figure 5**, social welfare response increases by 18.6% (from −8.67% point deviation in the pre-privatization economy to −10.28% point deviation in the post-privatization economy). Likewise, the privatization also brings a greater reduction in the social welfare level, as shown in panel B of **Figure 5**. When translated in original social welfare levels, the social welfare falls to −0.281 in the pre-privatization economy and to −0.248 in the post-privatization economy, which is quite close to the steady-state social welfare of the pre-privatization economy (−0.258 in **Table 2**). This reveals that potential social welfare gain from public pension privatization may be scraped away massively by one transitory negative aggregate shock.

Figure 3] Social Welfare Responses to a Negative TPF Shock:
Before and After Public Pension Privatization



4.3. Volatilities and Variations of Economies

So far, we just have illustrated a snapshot of economic and welfare responses to one event of a negative TPF shock before and after the privatization. However, such illustration alone is not sufficient for estimating the effect of public pension privatization on economic and welfare volatility (responsiveness) over the business cycle, because it is fully feasible that materialized aggregate productivity shock is positive or negative with magnitudes different from the one that we have examined right above. Thus, to properly estimate the effect of the privatization on economic and welfare responsiveness, we simulate series of 1000 TPF shocks with positive and negative shocks of various magnitudes, according to (6) and (19), and de-trend the growth rates of the after-shock 1000 aggregate variables of output, consumption, labor supply, investment, and social welfare, utilizing Hodrick–Prescott filter with smoothing parameter of 6.25 following Ravn and Uhlig (2002). Then, we obtain standard deviations of cyclic parts of these de-trended growth rates of aggregate output, consumption, labor supply, investment, and social welfare to measure volatilities of these variables before and after the privatization, which are reported in **Table 4**. On the other hand, having found that the privatization of PAYG public pension raises steady-state levels of aggregate labor supply and capital (as shown in **Table 2**), we also measure variation in aggregate labor supply responding the same 1000 TPF shocks to see whether we can utilize our estimation results for validating **Proposition 1** and **2** or not. To estimate the variation in not only total labor supply but also other macroeconomic variables, we also calculate standard deviations of cyclical parts of the de-trended variables themselves (instead of de-trended growth rates) and display the results in **Table 3**.

Table 3] Variations of Economies Before and After Public Pension Privatization

Public pension	$\left\|\sigma_Y\right\|$	$\left\|\sigma_C\right\|$	$\left\|\sigma_L\right\|$	$\left\|\sigma_K\right\|$	$\left\|\sigma_W\right\|$
PAYG	0.0190	0.0112	0.0019	0.1133	0.00011
	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)
Fully privatized	0.0175	0.0116	0.0014	0.1113	0.00012
	(91.9)	(103.6)	(73.7)	(98.2)	(103.4)

Note: $|\sigma_x|$ refers to the standard deviation of cyclical parts of the macroeconomic variable x, which is de-trended with Hodrick–Prescott filter. The first row reports the variation of the economy under the current PAYG Social Security pension system, whereas the second row does so for the economy after completion of privatizing the public pension. Numbers in parentheses refer to rescaled values of $|\sigma_x|$ with the pre-privatization value at 100 in order to express effects of the public pension privatization in percentage terms.

First of all, as shown in **Table 3**, we find that after the privatization of Social Security PAYG public pension, the variation (average size of level deviation from the steady state) of total output over the business cycle decreases substantially by 8.1% with reducing the variations of aggregate labor supply and capital by 26.3% and 1.8%, respectively, while variations of social welfare and aggregate consumption are increased by 3.4% and 3.6%, respectively. In particular, together with the increases in steady-state aggregate labor supply and capital caused by the privatization (**Table 2**), the decrease in variation of aggregate labor supply (**Table 3**) implies that we can apply **Proposition 1** and **2** for quantifying the trade-off between economic stability and social welfare volatility.

Table 4] Volatilities of Economies Before and After Public Pension Privatization

Public pension	σ_Y	$\frac{\sigma_C}{\sigma_Y}$	$\frac{\sigma_L}{\sigma_Y}$	$\frac{\sigma_K}{\sigma_Y}$	σ_W
PAYG	1.0828 (100.0)	0.9697 (100.0)	0.1271 (100.0)	2.5160 (100.0)	0.0414 (100.0)
Fully privatized	1.0417 (96.2)	1.0213 (105.3)	0.0943 (74.2)	2.3496 (93.4)	0.0492 (118.8)

Note: σ_x refers to the standard deviation of cyclical parts in the growth rates of the macroeconomic variable x, which is de-trended with Hodrick–Prescott filter. The first row reports the volatility of the economy under the current PAYG Social Security pension system, whereas the second row does so for the economy after completion of privatizing the public pension. Numbers in parentheses refer to rescaled values of σ_x with the pre-privatization value at 100 in order to express effects of the public pension privatization in percentage terms.

More importantly, estimated volatilities of the pre- and post- privatization economies, which are identical except for the PAYG public pension, are reported in **Table 4.**[14] We find that the privatization Social Security public pension enhances the stability of total output by 3.8% with decreasing the volatilities of aggregate investment and labor supply by 6.6% and 25.8%, respectively, while the privatization worsens the instability of social welfare by 18.8% with increasing the volatility of total consumption by 5.3% over the business cycle. Not surprisingly, these findings are in line with what we observe in **Figure 2** and **3** above. Rather, what is interesting is that the variations of aggregate output, labor supply, and investment still decline by the privatization (**Table 3**), despite the increases in steady-state total output, labor supply, and capital levels (**Table 2**), fortifying our finding that the privatization of PAYG

[14] Our simulation results in **Table 4** maintain the qualitative feature of US economy: rank of the volatilities (investment > output> labor supply> consumption).

public pension reduces economic volatility (as shown by the results in **Table 4**).

Above all, notice that our finding is consistent with **Proposition 1** and **2**, as the privatization decreases volatility of total output by reducing volatilities of aggregate labor supply and capital while raising instability of social welfare and total consumption over the business cycle. In other words, the privatization of PAYG public pension essentially generates a trade-off between macroeconomic stability and social welfare volatility.

After the privatization, wealth that individuals have accumulated for their retirement consumption is fully exposed to macroeconomic shocks; hence, their retirement wealth fluctuates more pro-cyclically than before the privatization. Without the PAYG public pension system that could have insured part of individuals' retirement wealth from macroeconomic risks over the business cycle, individuals now bear the full risks on their retirement wealth, which necessitates their additional efforts to secure stable provision of retirement consumption. Facing recession (expansion), due to more loss (gain) in their retirement wealth, individuals reduce (raise) their consumption more while decreasing (increasing) their labor supplies and savings with less degree. In turn, such decreases in volatilities of production inputs (labor and capital) eventually cause total output to go down (up) by less margin in the after-privatization economy than in the before-privatization economy.

Moreover, as individuals' utilities depend negatively on labor supply and positively on consumption, less reduction (less raise) in labor supply and more reduction (more raise) in consumption facing negative (positive) TPF shocks cause individuals to suffer (enjoy) more after the privatization, entailing social welfare to go down (up) further in the after-privatization economy than in the before-privatization economy.

Thus, by examining a dynamic aspect of economy, our study newly reveals a benefit of privatizing PAYG public pension — improvement in economic stability by decreases in volatilities of aggregate output, labor supply, and investment — which turns out not to be given for free but to entail a cost: increase in fluctuations of aggregate consumption and social welfare over the business cycle. Although how the government compares the benefit and the cost is not within the scope of our study, our finding suggests that the government needs to factor in these important benefit and cost which have been overlooked when assessing overall effects of the privatization.

5. CONCLUDING REMARKS

To investigate the impact of privatizing PAYG public pension on economic and welfare volatilities over the business cycle, we capitalize upon an overlapping generations model where heterogeneous individuals choose their own labor supplies and consumptions with idiosyncratic risks on their earning ability and longevity. By approximating optimal response to a TPF shock with log-linearization method, we theoretically prove that when privatization of PAYG public pension decreases variation in aggregate labor supply responding to TPF shocks with raising steady-state levels of aggregate labor supply and capital, the privatization reduces total output instability with improving stability of aggregate labor supply and investment at the cost of increasing volatilities of social welfare and total consumption over the business cycle. Without PAYG public pension, more exposure of their wealth for retirement consumption to negative (positive) TPF shocks leads individuals to reduce (increase) consumption more with decreasing (increasing) their labor supplies and saving less, which entails less reduction (increase) in output than before the privatization. In turn, reducing consumption more (less) and labor supply less (more) are translated into further increase (decrease) in individuals' utilities, which causes a rise in volatility of social welfare.

By calibrating parameters of our model to match data of US economy, we obtain stationary general equilibria of pre- and post-privatization economies and find that privatizing US PAYG public pension raises steady-state levels of total output, labor supply, and capital. We simulate the same series of 1000 TPF shocks on these two economies, which are identical except for the PAYG public pension, to estimate effect of the privatization on volatilities. We find that the public pension privatization reduces the volatilities of aggregate investment and labor supply by 6.6% and 25.8%, respectively, which causes total output to be less fluctuating by 3.8% over the business cycle. This gain of improved macroeconomic stability comes at a cost: increased instabilities of social welfare by 18.8% and of total consumption by 5.3%.

Above all, by embracing the reality that economy fluctuates, this study discovers an overlooked effect of privatizing PAYG public pension: a trade-off between macroeconomic and social welfare stabilities.

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APPENDIX

Proof for Proposition 1.

[step 0] For notational convenience, we add superscript p to describe the economy after public pension privatization. When the privatization of PAYG public pension decreases variation in aggregate labor supply responding to TPF shocks with raising steady-state levels of aggregate labor supply and capital, for an arbitrarily given TPF shock $\hat{z}_t = \hat{z}_t^p$ which equally hits both of pre- and post-privatization economies, $\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}^p \bar{l}_i^p(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s}) < 0$ with $\bar{L}^p - \bar{L} > 0$ and $\bar{K}^p - \bar{K} > 0$.

[step 1] By way of contradiction, suppose that aggregate labor supply volatility is not decreased. Then, $\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}^p(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}(\mathbf{s})dP(\mathbf{s}) \geq 0$. Notice that, due to (13), $\bar{L}^p - \bar{L} = \sum_{i=1}^{I}\int_{E\times K\times A} e\,\bar{l}_i^p(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} e\bar{l}_i(\mathbf{s})dP(\mathbf{s}) > 0$. This implies a contradiction to our assumption of $\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}^p \bar{l}_i^p(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s}) < 0$. Hence, $\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}^p(\mathbf{s}) dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}(\mathbf{s})dP(\mathbf{s}) < 0$, meaning that the privatization decreases volatility of aggregate labor supply when allowing TPF shocks to take various values over the business cycle.

[step 2] We want to show $\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A} \hat{k}_{t,i}^p\bar{k}_i^p(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A} \hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s}) < 0$ by way of contradiction. So, suppose that $\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A} \hat{k}_{t,i}^p\bar{k}_i^p(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A} \hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s}) \geq 0$.

By integrating (16), we get $\{\sum_{i=1}^{I}\int_{E\times K\times A} \hat{c}_{t,i}^p dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} \hat{c}_{t,i} dP(\mathbf{s})\} + E[\{\sum_{i=1}^{I}\int_{E\times K\times A} \hat{c}_{t+1,i+1}^p dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} \hat{c}_{t+1,i+1} dP(\mathbf{s})\}] = E[\hat{Y}_{t+1}^p - \hat{Y}_{t+1} - \{\frac{1}{\bar{K}^p}\sum_{i=1}^{I-1}\int_{E\times K\times A} \hat{k}_{t+1,i+1}^p\bar{k}_{i+1}^p(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I-1}\int_{E\times K\times A} \hat{k}_{t+1,i+1}\bar{k}_{i+1}(\mathbf{s}) dP(\mathbf{s})\}]$ since $\sum_{i=1}^{I}\int_{E\times K\times A} dP(\mathbf{s}) = 1$. Lagging this by one period, we get $\{\sum_{i=1}^{I+1}\int_{E\times K\times A} \hat{c}_{t-1,i-1}^p dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A} \hat{c}_{t-1,i-1} dP(\mathbf{s})\} + \{\sum_{i=1}^{I+1}\int_{E\times K\times A} \hat{c}_{t,i}^p dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A} \hat{c}_{t,i} dP(\mathbf{s})\} = \hat{Y}_t^p - \hat{Y}_t - \{\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A} \hat{k}_{t,i}^p$

$\bar{k}_i^{\,p}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s})\}$. Let us this lagged integrated (16) be labeled as (16)′.

Moreover, by integrating (15), $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}(\mathbf{s})dP(\mathbf{s}) = \hat{Y}_t^{p} - \hat{Y}_t - \{\frac{1}{\bar{L}^p}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}\bar{l}_i^{p}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s})\} - \frac{1}{\eta}\{\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}(\mathbf{s})dP(\mathbf{s})\}$, which is labeled as (15)′. By integrating (18), as $\hat{z}_t = \hat{z}_t^{p}$, $\hat{Y}_t^{p} - \hat{Y}_t = \alpha\{\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\bar{k}_i^{\,p}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s})\} + (1-\alpha)\{\frac{1}{\bar{L}^p}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}\bar{l}_i^{p}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s})\}$. Let us label this as (18)′.

Combining (15)′ and (18)′, we get $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}(\mathbf{s})dP(\mathbf{s}) = \alpha\{\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\bar{k}_i^{\,p}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s})\} - \alpha\{\frac{1}{\bar{L}^p}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}\bar{l}_i^{p}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s})\} - \frac{1}{\eta}\{\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}(\mathbf{s})dP(\mathbf{s})\}$. The terms within the second and third parentheses are negative due to [step 1], $\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}\bar{l}_i^{p}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s}) < 0$ and $\bar{L}^p - \bar{L} > 0$. As supposed at the beginning of [step 2], the first parenthesis is positive. Taking these together, as $\alpha > 0$ and $\eta > 0$, $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}(\mathbf{s})dP(\mathbf{s}) > 0$.

This implies that the left hand side of (16)′ $\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t-1,i-1}^{p}dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t-1,i-1}dP(\mathbf{s})\} + \{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t,i}^{p}dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t,i}dP(\mathbf{s})\} > 0$, because no one alive is of age $I+1$ and age i starts from 1 in our model means that $\hat{c}_{t-1,0}^{p} = \hat{c}_{t-1,0} = 0 = \hat{c}_{t-1,I+1}^{p} = \hat{c}_{t-1,I+1}$.

Combining (16)′ and (18)′, we obtain $\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t-1,i-1}^{p}dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t-1,i-1}dP(\mathbf{s})\} + \{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t,i}^{p}dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t,i}dP(\mathbf{s})\} = (\alpha-1)\{\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\bar{k}_i^{\,p}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s})\} + (1-\alpha)\{\frac{1}{\bar{L}^p}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}\bar{l}_i^{p}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s})\}$. Since $1 > \alpha > 0$, the left hand side of this equation are negative, which means that the right hand side of (16)′ is negative: A contradiction. This proves that $\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\bar{k}_i^{\,p}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s}) < 0$.

Furthermore, since $\bar{K}^p - \bar{K} = \sum_{i=1}^{I}\int_{E\times K\times A}\bar{k}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\bar{k}_{t,i}(\mathbf{s})dP(\mathbf{s}) > 0$, this implies that

$\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}(\mathbf{s})dP(\mathbf{s})<0$. Therefore, volatility of aggregate investment is decreased by the privatization, when allowing TPF shocks to take various values over the business cycle.

[step 3] From [step 1] and [step 2], we prove that $\{\frac{1}{\overline{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\overline{k}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\overline{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\overline{k}_{i}(\mathbf{s})dP(\mathbf{s})\}<0$ and $\frac{1}{\overline{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}\overline{l}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\overline{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\overline{l}_{i}(\mathbf{s})dP(\mathbf{s})<0$. Due to (18)′ and $1>\alpha>0$, $\hat{Y}_{t}^{p}-\hat{Y}_{t}<0$, which means that the privatization decreases volatility of total output when allowing TPF shocks to take various values over the business cycle. **Q.E.D**

Proof for Proposition 2.

[step 0] At the outset, let us explicitly state volatility of social welfare. Since individuals' utilities are at their maximums at steady state, before any shock hits, social welfare is $\sum_{i=1}^{I}\int_{E\times K\times A}V_{i}(\mathbf{s})dP(\mathbf{s})=\sum_{i=1}^{I}\int_{E\times K\times A}u(\overline{c}_{i},\overline{l}_{i})+m_{i}\beta E[V_{i+1}(\mathbf{s})]dP(\mathbf{s})$. Moreover, when a TPF shock hits, response of each individual utility are realized via deviations of their current labor supply and consumption from their steady-state own levels. Therefore, the entailed change in utility brought by the TPF shock is $u(c_{t,i},l_{t,i})+m_{i}\beta E[V_{i+1}(\mathbf{s})]-\{u(\overline{c}_{i},\overline{l}_{i})+m_{i}\beta E[V_{i+1}(\mathbf{s})]\}=u(\overline{c}_{i}\exp(\hat{c}_{t,i}),\overline{l}_{i}\exp(\hat{l}_{t,i}))-u(\overline{c}_{i},\overline{l}_{i})$. Notice that volatility is measured in relative term, as $100\hat{l}_{t,i}$ is % deviation from $\overline{l}_{i}$. Likewise, the volatility of social welfare is measured by $s\hat{w}_{t}=sign(\sum_{i=1}^{I}\int_{E\times K\times A}V_{i}(\mathbf{s})dP(\mathbf{s}))\sum_{i=1}^{I}\int_{E\times K\times A}\hat{v}_{t,i}dP(\mathbf{s})$ where $\hat{v}_{t,i}=\frac{u(c_{t,i},l_{t,i})-u(\overline{c}_{i},\overline{l}_{i})}{\sum_{i=1}^{I}\int_{E\times K\times A}u(\overline{c}_{i},\overline{l}_{i})+m_{i}\beta E[V_{i+1}(\mathbf{s})]dP(\mathbf{s})}$; and $sign(x)$ is a function of a real number x that returns the sign of x by $\frac{x}{|x|}$. Because the entailed change in social welfare, $u(c_{t,i},l_{t,i})-u\ (\overline{c}_{i},\overline{l}_{i})$, takes opposite sign when it is divided by a negative $\sum_{i=1}^{I}\int_{E\times K\times A}V_{i}(\mathbf{s})dP(\mathbf{s})$. Thus, in order to maintain the original change, regardless of the sign of $\sum_{i=1}^{I}\int_{E\times K\times A}V_{i}(\mathbf{s})dP(\mathbf{s})$, we multiply $sign(\sum_{i=1}^{I}\int_{E\times K\times A}V_{i}(\mathbf{s})dP(\mathbf{s}))$. Moreover, because the after-shock deviation is approximated in the neighborhood of steady-state, the sign of social welfare stays the same. That is, $sign(\sum_{i=1}^{I}\int_{E\times K\times A}u(\overline{c}_{i},\overline{l}_{i})+m_{i}\beta E[V_{i+1}(\mathbf{s})]dP(\mathbf{s}))$ =$sign(\sum_{i=1}^{I}\int_{E\times K\times A}u(c_{t,i},l_{t,i})+m_{i}\beta E[V_{i+1}(\mathbf{s})]dP(\mathbf{s}))$.

[step 1] For notational convenience, we add superscript p to describe the economy after public pension privatization. When the privatization of PAYG public pension decreases variation in aggregate labor supply responding to TPF shocks with raising steady-state levels of aggregate labor supply and capital, for an arbitrarily given TPF shock $\hat{z}_{t}=\hat{z}_{t}^{p}$ which

equally hits both of pre- and post-privatization economies, $\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}^{p}\bar{l}_i^{p}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s}) < 0$ with $\bar{L}^p - \bar{L} > 0$ and $\bar{K}^p - \bar{K} > 0$. According to the step 2 of proof for **Proposition 1**, $\sum_{i=1}^{I}\int_{E\times K\times A} \hat{c}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} \hat{c}_{t,i}(\mathbf{s})dP(\mathbf{s}) > 0$ which implies that volatility of aggregate consumption is increased by the privatization. At the same time, as **Proposition 1** shows, volatilities of aggregate output and labor supply are decreased by the privatization.

[step 2] We first examine whether increase in consumption volatility raises social welfare volatility or not. To this end, we need to check the sign of $\frac{ds\hat{w}_t}{d\hat{c}_{t,i}}$. Notice that $\frac{ds\hat{w}_t}{d\hat{c}_{t,i}} = \frac{ds\hat{w}_t}{d\hat{v}_{t,i}}\frac{d\hat{v}_{t,i}}{dc_{t,i}}\frac{dc_{t,i}}{d\hat{c}_{t,i}} + \frac{ds\hat{w}_t}{d\hat{v}_{t,i}}\frac{d\hat{v}_{t,i}}{d\bar{c}_i}\frac{d\bar{c}_i}{d\hat{c}_{t,i}} = \frac{ds\hat{w}_t}{d\hat{v}_{t,i}}[\frac{d\hat{v}_{t,i}}{dc_{t,i}}\frac{dc_{t,i}}{d\hat{c}_{t,i}} + \frac{d\hat{v}_{t,i}}{d\bar{c}_i}\frac{d\bar{c}_i}{d\hat{c}_{t,i}}]$. Firstly, $\frac{d\hat{v}_{t,i}}{dc_{t,i}}\frac{dc_{t,i}}{d\hat{c}_{t,i}} + \frac{d\hat{v}_{t,i}}{d\bar{c}_i}\frac{d\bar{c}_i}{d\hat{c}_{t,i}} = \frac{u_c(\bar{c}_i \exp(\hat{c}_{t,i}))}{\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})} + \{\frac{u_c\sum_{i=1}^{I}\int_{E\times K\times A} u(c_{t,i},l_{t,i}) + m_i\beta E[V_{i+1}(\mathbf{s})]dP(\mathbf{s})}{[\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})]^2}\}(\frac{c_{t,i}}{\exp(\hat{c}_{t,i})})$. Since $u_c > 0$ and $sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})) = sign(\sum_{i=1}^{I}\int_{E\times K\times A} u(c_{t,i},l_{t,i}) + m_i\beta E[V_{i+1}(\mathbf{s})]dP(\mathbf{s}))$, the sign of $\frac{d\hat{v}_{t,i}}{dc_{t,i}}\frac{dc_{t,i}}{d\hat{c}_{t,i}} + \frac{d\hat{v}_{t,i}}{d\bar{c}_i}\frac{d\bar{c}_i}{d\hat{c}_{t,i}}$ is equal to $sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))$. Secondly, the sign of $\frac{ds\hat{w}_t}{d\hat{v}_{t,i}}$ is equal to $sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))$. Taken together, $sign(\frac{ds\hat{w}_t}{d\hat{c}_{t,i}}) = sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})) > 0$, which implies that the increase in aggregate consumption volatility from [step 1] raises social welfare volatility; i.e., $s\hat{w}_t^p - s\hat{w}_t > 0$.

[step 3] Next, we examine whether decrease in volatility of labor supply raises social welfare volatility or not by finding the sign of $-\frac{ds\hat{w}_t}{d\hat{l}_{t,i}}$. Notice that $\frac{ds\hat{w}_t}{d\hat{l}_{t,i}} = \frac{ds\hat{w}_t}{d\hat{v}_{t,i}}[\frac{d\hat{v}_{t,i}}{dl_{t,i}}\frac{dl_{t,i}}{d\hat{l}_{t,i}} + \frac{d\hat{v}_{t,i}}{d\bar{l}_i}\frac{d\bar{l}_i}{d\hat{l}_{t,i}}]$. Firstly, $\frac{d\hat{v}_{t,i}}{dl_{t,i}}\frac{dl_{t,i}}{d\hat{l}_{t,i}} + \frac{d\hat{v}_{t,i}}{d\bar{l}_i}\frac{d\bar{l}_i}{d\hat{l}_{t,i}} = \frac{1}{[\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})]^2}\{u_l(\bar{l}_i \exp(\hat{l}_{t,i}))\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s}) dP(\mathbf{s}) + (\frac{u_l l_{t,i}}{\exp(\hat{l}_{t,i})})(\sum_{i=1}^{I}\int_{E\times K\times A} u(c_{t,i},l_{t,i}) + m_i\beta E[V_{i+1}(\mathbf{s})]dP(\mathbf{s}))\}$. Because $u_l < 0$ and $sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})) = sign(\sum_{i=1}^{I}\int_{E\times K\times A} u(c_{t,i},l_{t,i}) + m_i\beta E[V_{i+1}(\mathbf{s})]dP(\mathbf{s}))$, the sign of $\frac{d\hat{v}_{t,i}}{dl_{t,i}}\frac{dl_{t,i}}{d\hat{l}_{t,i}} + \frac{d\hat{v}_{t,i}}{d\bar{l}_i}\frac{d\bar{l}_i}{d\hat{l}_{t,i}}$ is equal to $-sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))$. Secondly, since the sign of $\frac{ds\hat{w}_t}{d\hat{v}_{t,i}}$ is equal to

$sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))$, $sign(-\frac{\mathrm{d}s\hat{w}_t}{\mathrm{d}\hat{l}_{t,i}}) = sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})) > 0$. As a consequence, the decrease in volatility of aggregate labor supply from [step 1] raises social welfare volatility as well.

[step 4] Finally, [step 1], [step 2], and [step 3] together imply that the privatization increases volatility of social welfare, while decreasing volatility of total output and increasing volatility of aggregate consumption (according to **Proposition 1**). **Q.E.D**