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Public Pension Privatization and Economic Volatility over the Business Cycle

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Using an overlapping-generations model where heterogeneous individuals choose their consumptions and labor supplies for responding to aggregate shocks, we theoretically show that privatizing pay-as-you-go public pension can decrease the volatilities of aggregate output, labor supply, and investment, while increasing the volatilities of total consumption and social welfare. Via simulations with US data, we find that privatizing Social Security enhances the stability of total output by 3.8% with decreasing the volatilities of aggregate labor supply and investment by 28.6% and 10.7%, respectively, while increasing the volatilities of social welfare and total consumption by 18.8% and 1.3%, respectively.(JEL Code: H55, E62, E32)

Keywords: public pension privatization, economic stability, social welfare volatility, responsiveness to TPF shocks

I. INTRODUCTION

Facing rapid rises in longevity, pay-as-you-go (PAYG) public pension system becomes fiscally nonviable in many countries.[1] Among various measures proposed to reform unsustainable PAYG public pension system, privatizing public pension draws most of attentions and is actually implemented in some countries. While solving the fiscal insolvency problem, privatization of PAYG public pension may affect economic responsiveness to aggregate shocks, because it will change individuals' labor supply and savings responses to the shocks by raising the exposure of their retirement wealth to the shocks. Since public pension funds are quite sizable (e.g., 17.9% of GDP

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[1] For example, among the OECD countries, the average public pension debt is projected to reach 124.2% of GDP by 2030 (Disney, 2000). Moreover, in the United States, funds for the Social Security retirement program will be depleted by 2033.

in the US),[2] the impact of public pension privatization would be neither trivial nor limited to a small part of economy. Nevertheless, little is studied on how public pension privatization affects the degree of economic fluctuations. This paper aims to investigate the effect of privatizing PAYG public pension on the volatilities of economy and social welfare in responding to aggregate productivity shocks over the business cycle.

To assess the effects of privatizing PAYG public pension, most of the previous studies have examined only steady-state equilibria before and after the privatization (e.g., Hubbard and Judd 1987; Breyer and Straub 1993; İmrohoroğlu et al. 2003; Nishiyama and Smetters 2007; Fuster et al. 2007).[3] However, such steady-state analyses leave us ignorant of how privatizing PAYG public pension affects the degree of economic fluctuations responding to aggregate shocks that entail deviations from the steady state. Krueger and Kubler (2006) and Olovsson (2010) embraced aggregate shocks by allowing total factor productivity (TFP) to take four different levels; nevertheless, they did not analyze effect of privatizing PAYG public pension on the volatilities of social welfare or aggregate economic variables. Moreover, Krueger and Kubler (2006) and Olovsson (2010) did not allow individuals to choose their own labor supply, which limits the applicability of their works since public pension privatization clearly makes a difference in labor supply decisions.

For improving upon the previous studies, we derive individuals' optimal responses to a TFP shock, based on an overlapping generations model where heterogeneous individuals choose their own labor supplies and consumptions, to theoretically prove that privatizing PAYG public pension can improve the stability of total output by reducing the volatilities of aggregate labor supply

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[2] This is also similar to the average for the countries in the Organization for Economic Co-operation and Development: 19.6% of GDP (OECD, 2013).

[3] Hubbard and Judd (1987), Krueger and Kubler (2006), and İmrohoroğlu et al. (2003) regarded an economy without public pension as pre-reform initial state and examined stationary equilibrium after introducing PAYG public pension system. In contrast, Nishiyama and Smetters (2007), and Fuster et al. (2007) treated economy with PAYG public pension system as initial state. Either way, these studies agree in the finding that privatizing PAYG public pension increases steady-state capital and labor supply (and thus output), although they disagree in overall welfare outcome of the privatization.

and investment, while increasing the volatilities of social welfare and total consumption over the business cycle. In other words, our study newly reveals an overlooked effect of public pension privatization: a trade-off between macroeconomic stability and social welfare volatility.

Privatizing PAYG public pension causes individuals' retirement wealth to fluctuate more pro-cyclically by increasing its exposure to macroeconomic shocks. This generates the incentive of precautionary savings as well as additional wealth effects on individuals' responses of labor supply and consumption. When a negative (positive) aggregate shock is materialized, facing greater loss (gain) in their retirement wealth without PAYG public pension insurance, individuals decrease (increase) their labor supplies and savings by *smaller* margin than before the public pension privatization, while they reduce (raise) their consumptions by *larger* margin. In turn, the ensuing decreases in the volatilities of aggregate labor supply and investment entail a decrease in total output instability over the business cycle. However, such a beneficial effect of the public pension privatization — improvement in economic stability — turns out not free of side effect. That is, the volatilities of social welfare and total consumption are increased by the privatization, since individuals end up with suffering (enjoying) more by working more (less) and consuming less (more), for responding to the negative (positive) aggregate shock, than before the privatization.

To quantify our theoretical findings, we calibrate our model to match US economy data and obtain stationary general equilibria before and after privatizing Social Security PAYG public pension. By introducing the same series of 1000 TFP shocks to the two steady-state economies that are identical except for the Social Security public pension, we find that the public pension privatization reduces the instability of total output by 3.8% with decreasing the volatilities of aggregate labor supply and investment by 28.6% and 10.7%, respectively. At the same time, the privatization also raises the volatilities of social welfare and total consumption by 18.8% and 1.3%, respectively, over the business cycle.

The rest of this paper unfolds as follows. Section II describes an overlapping-generations model from which we derive theoretical property regarding impact of privatizing PAYG public pension on the volatilities of economy and social welfare. For empirical analysis, Section III calibrates our model to the US economy to simulate aggregate productivity shocks. Section IV estimates the effect of privatizing Social Security public pension on the volatilities of macroeconomic variables and social welfare. Section V concludes the paper.

II. MODEL

Consider an economy that is inhabited by heterogeneous individuals who are different in age i and earning ability e. Each period t, individuals are born with zero endowment and none of them is alive beyond age of I (i.e., $i \in \{1, \cdots, I\}$). Thus, at any given time, I different age cohorts co-exist in the economy. Living age-i individuals survive to be of age $i+1$ with probability m_i, while the total population grows at a constant rate n. Moreover, for each period, individuals' earning ability $e \in E = [\underline{e}, \overline{e}] > 0$ is subject to idiosyncratic shocks.

Each period t, given the state of the economy and government policies, new-born age-1 individuals choose their consumptions, labor supplies, and savings (investments) of their life time by solving the following maximization problem:

$$max \sum_{i=1}^{I} E[\beta^{i-1}(\prod_{h=1}^{i} m_h) u(c_{t+i-1,i}, l_{t+i-1,i})], \qquad (1)$$

where $c_{t,i}$ and $l_{t,i}$ are consumption and labor supply, respectively, at age i in period t; β is the time preference parameter. The amount of time given to every individual for each period is one; thus, $l_{t,i} \in [0,1]$ for $\forall t$ and $\forall i$. Moreover, for the within-period utility function $u(c_{t,i}, l_{t,i})$, we use

$$u(c_{t,i}, l_{t,i}) = \log(c_{t,i}) - \psi \frac{l_{t,i}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}. \qquad (2)$$

As you may notice, the coefficient of relative risk aversion is set to 1, following Chetty (2006) that estimates the parameter based on empirical findings on observed labor supply behavior. This is lower than the values used in the previous studies such as 4 in Fuster et. al (2007) or 2 in Nishiyama and Smetters (2007) which are chosen without considering consistency with any empirical finding on labor supply behavior.

In addition, when solving (1), each individual faces intertemporal budget constraints as follows: for $\forall i$,

$$w_{t+i-1} e l_{t+i-1,i}(1-\tau_p-\tau_l) + (1+(1-\tau_k)r_{t+i-1})k_{t+i-1,i} + q_{t+i-1} + 1_R(i)a_{t+i-1,i} \geq c_{t+i-1,i} + (1+g)k_{t+i,i}, \qquad (3)$$

where w_t and r_t are the market wage rate and interest rate, respectively, of period t; τ_p is the contribution rate for public pension whereas τ_l and τ_k are the rates of taxes on labor income and capital gain, respectively; q_t is accidental bequest from those who die in period t; g is the growth rate of this economy. Moreover, $1_R(i)$ is a binary indicator for whether an age-i individual is eligible to receive public pension benefit (taking the value of one if he is eligible or zero otherwise); and the amount of public pension benefit for him in period t is $a_{t,i}$, whose calculation formula is specified by the government. While $k_{t+1,i}$ is chosen level of savings by an age-i individual in the current period t, $k_{t,i}$ is the investment (savings) made by him in the previous period. Individuals face a borrowing constraint for each period; hence, $k_{t+i-1,i} \geq 0$ for $\forall i$. Furthermore, insurances for earning ability and mortality are not provided in the private market.

For simplicity, bequest that is left accidentally by individuals who do not survive to the next period is not gone with the deceased but distributed equally to surviving individuals, as in previous studies like Nishiyama and Smetters (2007) and Fuster et al. (2007). That is, for $\forall t$,

$$q_t = \sum_{i=1}^{I}(1-m_i)\int_{E\times K\times A} k_{t+1,i}(\mathbf{s}_t)dP_t(\mathbf{s}_t)\{\sum_{i=1}^{I} m_i \int_{E\times K\times A} dP_t(\mathbf{s}_t)\}^{-1}, \qquad (4)$$

where $\mathbf{s}_t = (i, e, k_{t,i}, a_{t,i})$ is a vector of state variables at the beginning of period t; the sets of E, K, and A denote the supports for earning ability, wealth (capital), and potential amount of public pension benefit, respectively.[4] The distribution of the state variables $P_t(\mathbf{s}_t)$ evolves as follows: for $\forall t$,

$$p_{t+1}(i+1, e, k', a') = \frac{m_i}{1+n}\int_{E\times K\times A} 1(k' = k_{t+1,i}(\mathbf{s}_t) + q_t)\times 1(a' = a_{t+1,i+1}(w_t e l_{t,i}(\mathbf{s}_t),$$
$$a_{t,i}))dP_t(\mathbf{s}_t), \qquad (5)$$

where $p_{t+1}(\mathbf{s}_{t+1})$ refers to the probability density of the state variables $\mathbf{s}_{t+1}$; $1(\cdot)$ is an indicator function that takes the value of one if the statement inside the parenthesis is true and the value of zero otherwise. Moreover, $k_{t+1,i}(\mathbf{s}_t)$ and $l_{t,i}(\mathbf{s}_t)$ are decision rules (policy functions) of savings and labor supply, respectively, obtained from age-i individuals' maximizing their utility for their remaining lifetime in period t.

In brief, for any given i, the decision problem that an age-i individual solves for period t is written as

$$V_i(\mathbf{s}_t) = max\, u(c_{t,i}, l_{t,i}) + m_i\beta E[V_{i+1}(\mathbf{s}_{t+1})], \qquad (6)$$

where $V_i(\cdot)$ is the maximized utility function (value function) of an age-i individual for the remaining lifetime subject to the intertemporal budget constraint (3), (4), (5), and the government policies.

4 Even if an age-*i* individual is not yet eligible to receive the public pension benefit in period *t* because he is younger than the public pension entitlement age (i.e., $1_R(i) = 0$), the public pension contributions that he has made can determine the amount of "potential" public pension benefit $a_{t,i}$, according to the given benefit calculation formula.

In addition, there exists a representative firm in the economy which produces output Y_t for period t, following a Cobb-Douglas technology as below.

$$Y_t = F(K_t, L_t) = z_t K_t^{\alpha} L_t^{1-\alpha}, \quad (7)$$

where K_t is aggregate capital input invested by individuals in the economy and L_t is the total sum of individuals' labor supplies in efficiency units. In particular, z_t is TFP that is subject to a random shock as

$$z_{t+1} = z_t^{\rho} \exp(\varepsilon_{t+1}) \text{ where } \varepsilon_t \sim WN(0, \sigma_z^2) \text{ for } \forall t. \quad (8)$$

The firm solves its profit maximization problem of $\underset{\{K_t, L_t\}}{max} z_t K_t^{\alpha} L_t^{1-\alpha} - (r_t + \delta)$ $K_t - w_t L_t$, which yields the following decision rules: for $\forall t$,

$$F_K(K_t, L_t) = r_t + \delta \quad (9)$$

$$F_L(K_t, L_t) = w_t \quad (10)$$

where δ is a depreciation rate of capital.

The government chooses labor income tax rate τ_l to balance its budget as follows.

$$G + \sum_{i=R}^{I} \int_{E \times K \times A} a_{t,i}(\mathbf{s}_t) dP_t(\mathbf{s}_t) = \sum_{i=1}^{I} \int_{E \times K \times A} \{w_t e l_{t,i}(\mathbf{s}_t)[\tau_l + \tau_p] + \tau_k r_t k_{t,i}(\mathbf{s}_t)\} dP_t(\mathbf{s}_t), \quad (11)$$

where G is a given required government expenditure, as in the previous studies, and R is the public pension entitlement age from which an individual is eligible to receive fully vested public pension benefit $a_{t,i}$. Whereas individuals cannot receive the public pension benefit $a_{t,i}$ before the age of R, they cannot delay or stop receiving the benefit after R. That is,

$$1_R(i) = \begin{cases} 0 \text{ if } i < R \\ 1 \text{ if } i \geq R \end{cases}. \quad (12)$$

Taking all the three parties (individuals, firm, and government) together, this economy as a whole meets the following aggregate resource constraint

$$Y_t = C_t + (1+n)(1+g)K_{t+1} - (1-\delta)K_t + G + \sum_{i=R}^{I} \int_{E\times K\times A} a_{t,i}(\mathbf{s}_t)dP_t(\mathbf{s}_t), \qquad (13)$$

where $C_t \equiv \sum_{i=1}^{I} \int_{E\times K\times A} c_{t,i}(\mathbf{s}_t)dP_t(\mathbf{s}_t)$ is aggregate consumption. Via competitive markets where individuals and the firm solve their own maximization problems, this economy reaches a stationary general equilibrium that is defined as below.

Given the government policies $\{G, R, \tau_p, \tau_k\}$ and the public pension benefit formula, the stationary recursive competitive equilibrium of this economy is a set of value functions $\{V_i(\mathbf{s}_t)\}_{i=1}^{I}$; decision rules $\{c_{t,i}(\mathbf{s}_t)$ $, l_{t,i}(\mathbf{s}_t), k_{t+1,i}(\mathbf{s}_t), a_{t,i}(\mathbf{s}_t)\}_{i=1}^{I}$; the associated distribution of the state variables defined by its probability measure $p_t(\mathbf{s}_t)$ following (5); a lump-sum transfer of accidental bequest q_t; labor income tax rate τ_l; and the prices of labor w_t and capital r_t that satisfy the following conditions for $\forall t$:

(i) Given the government policies, factor prices, and transfer of accidental bequest, all individuals' decision rules solve their own problem of (6).

(ii) The representative firm maximizes its profit by satisfying (9) and (10) with the factor markets being cleared as below:

$$K_t = \sum_{i=1}^{I} \int_{E\times K\times A} k_{t+1,i}(\mathbf{s}_t)dP_t(\mathbf{s}_t) \qquad (14)$$

$$L_t = \sum_{i=1}^{I} \int_{E\times K\times A} el_{t,i}(\mathbf{s}_t)dP_t(\mathbf{s}_t), \qquad (15)$$

which satisfy (13) as well.

(iii) The government sets τ_l according to (11) and equally distributes q_t that is defined by (4).

(iv) This economy reaches a steady state by meeting

$$p_t(\mathbf{s}) = p_{t+1}(\mathbf{s}) \text{ for } \forall \mathbf{s} \in \{1, \cdots, I\} \times E \times K \times A \text{ and } \forall t. \qquad (16)$$

In a steady state, therefore, all the variables drop their time subscript.

Moreover, social welfare (weighted sum of the utilities of all individuals) at the equilibrium is $\sum_{i=1}^{I} \int_{E \times K \times A} V_i(\mathbf{s}) dP(\mathbf{s})$.

Having obtained a steady-state general equilibrium, we further allow this economy to fluctuate, instead of staying steadfastly at the steady state, by letting this economy respond to macroeconomic shocks. In our model, an aggregate shock is described as an unexpected change in TFP z_t, which propagates throughout the entire economy via reactions of all individuals and the firm. According to Uhlig (1999), we approximate the equilibrium laws of such reactions by log-linearization. From intra- and inter-temporal conditions for the individuals' dynamic optimization of (6) with respect to labor supply, consumption, and savings, we derive the log-linearized optimal response as follows:

$$\frac{1}{\eta} e\hat{l}_{t,i} = \hat{Y}_t - \frac{1}{\bar{L}} \left(\sum_{i=1}^{I} \int_{E \times K \times A} e\hat{l}_{t,i} \bar{l}_i(\mathbf{s}) dP(\mathbf{s}) \right) - \hat{c}_{t,i} \quad \text{for} \quad \forall i \in \{1, \cdots, I\} \qquad (17)$$

$$\hat{c}_{t,i} = E[\hat{Y}_{t+1} - \frac{1}{\bar{K}} \sum_{i=1}^{I-1} \int_{E \times K \times A} \hat{k}_{t+1,i+1} \bar{k}_{i+1}(\mathbf{s}) dP(\mathbf{s}) - \hat{c}_{t+1,i+1}] \quad \text{for} \quad \forall i \in \{1, \cdots, I-1\} \qquad (18)$$

$$(\hat{Y}_t - \frac{1}{\bar{L}} \sum_{i=1}^{I} \int_{E \times K \times A} e\hat{l}_{t,i} \bar{l}_i(\mathbf{s}) dP(\mathbf{s}) + \hat{l}_{t,i}) \bar{w} e\bar{l}_i (1 - \tau_l - \tau_p) + (1 - \tau_k) \bar{r} \bar{k}_i (\hat{Y}_t - \frac{1}{\bar{K}} \sum_{i=1}^{I} \int_{E \times K \times A} \hat{k}_{t,i} \bar{k}_i(\mathbf{s})$$
$$dP(\mathbf{s})) + (1 + (1 - \tau_k) \bar{r}) \bar{k}_i \hat{k}_{t,i} = \bar{c}_i \hat{c}_{t,i} + (1 + g) \bar{k}_{i+1} \hat{k}_{t+1,i+1} \quad \text{for} \quad \forall i \in \{1, \cdots, I\}, \qquad (19)$$

where $\bar{x}$ is a steady-state value of variable x_t and $\hat{x}_t = \log(\frac{x_t}{\bar{x}})$. As Uhlig (1999) shows, when the after-shock value of x_t lies in the neighborhood of its before-shock value of $\bar{x}$, $\hat{x}_t$ captures volatility (responsiveness) of the variable x_t since $100\hat{x}_t$ approximates % deviation of the variable x_t from its steady-state value $\bar{x}$. In this line, variation (level-deviation from its steady-state value) of the variable x_t is measured as $\hat{x}\bar{x}$. From the representative firm's production (7) and (8), we also obtain the following equilibrium laws of motion by which a TFP shock spreads throughout the entire economy.

$$\hat{Y}_t = \hat{z}_t + \frac{\alpha}{\bar{K}} \sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i} \bar{k}_i(\mathbf{s}) dP(\mathbf{s}) + \frac{1-\alpha}{\bar{L}} \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i} \bar{l}_i(\mathbf{s}) dP(\mathbf{s}) \qquad (20)$$

$$\hat{z}_t = \rho \hat{z}_{t-1} + \varepsilon_t. \qquad (21)$$

After all, by introducing a TFP shock with an unexpected change in ε_t of (21), which is initially zero at the steady-state, and then by solving a system of the above linear equations (17), (18), (19), (20), and (21), we obtain economic and welfare responses to the exogenous aggregate productivity shock.

From our general model, we can derive theoretical statements about the effect of public pension privatization on the volatilities of macroeconomic variables and social welfare as below.

Proposition 1. When privatizing PAYG public pension increases steady-state levels of aggregate labor supply and capital and decreases variation in aggregate labor supply from responding to a TFP shock, the privatization decreases the volatility of total output over the business cycle by reducing the volatilities of aggregate labor supply and investment.

Proof. See Appendix.

Clearly, improvement in economic stability from decreases in the volatilities of total output, labor supply, and investment is a beneficial effect of PAYG public pension privatization. However, it turns out not free of side effects as the privatization simultaneously increases the volatilities of social welfare and total consumption.

Proposition 2. When privatizing PAYG public pension increases steady-state levels of aggregate labor supply and capital and decreases variation in aggregate labor supply from responding to a TFP shock, the privatization increases the volatilities of social welfare and total consumption over the business cycle while reducing the volatility of total output.

Proof. See Appendix.

As a matter of fact, various previous studies (e.g., Hubbard and Judd 1987; Breyer and Straub 1993; İmrohoroğlu et al. 2003; Nishiyama and Smetters 2007; Fuster et al. 2007) have proven that privatizing PAYG public pension raises steady-state levels of aggregate labor supply and capital, since the privatization removes labor supply distortions brought by the payroll taxes for public pension contribution, while it nullifies the crowding-out effect of PAYG public pension on savings. Without public pension benefits, to secure the resource for their retirement consumption, individuals save more than before the public pension privatization.

Privatizing (i.e., eliminating) PAYG public pension insurance exposes full of individuals' retirement wealth to aggregate shocks. Thus, facing a negative (positive) macroeconomic shock, individuals experience greater loss (gain) in their retirement wealth, so they reduce (increase) their labor supplies by *smaller* margin and their consumptions by *larger* margin than before the privatization. Furthermore, when such negative (positive) wealth effect from the increased exposure of retirement wealth to the aggregate shock is large enough to decrease the level-deviation of labor supply from its steady-state value for responding to the shock, despite the increased steady-state level of wealth (capital) held, the individuals also reduce (increase) their savings by *smaller* margin than before the privatization. As a result, the privatization decreases the volatilities of aggregate labor supply, and investment, which causes the volatility of total output to fall. At the same time, the privatization increases the volatility of social welfare, because individuals suffer (enjoy) more from more (less) labor and less (more) consumption facing the negative (positive) shock in the after-privatization economy than in the before-privatization economy.

To quantify our theoretical findings of the trade-off between economic stability and social welfare volatility, in the next section, we calibrate our model to the US data for estimating the effect of privatizing the Social Security PAYG public pension on the volatilities of macroeconomic variables and social welfare over the business cycle.

III. CALIBRATION

We calibrate our overlapping generations model to match the United States economy under the current Social Security retirement program, as our baseline (pre-reform) economy. One period in our model is equivalent to one year. Individuals of age 1 ($i = 1$) in our model correspond to 21-year-old individuals. The sequence of survival rates (m_i) is obtained from the data of life tables released by the United States Centers for Disease Control and Prevention, based on which we set $I = 80$.

For the Frisch elasticity of labor supply which is η, we select the value of 1.5, because Chang and Kim (2014) proved this value to well match real-data volatility of labor hours.[5] Fiorito and Zanella (2012) found that estimates for Frisch elasticity consistent with the observed volatility in aggregate labor supply range from 1.1 to 1.7. Moreover, the model that Chang and Kim (2014) used is closer to our model than any others like Cho and Cooley (1994) or Imai and Keane (2004).[6] Although no predominant consensus on the value of Frisch elasticity is yet established, İmrohoroğlu and Kitao (2009) showed that effect of public pension privatization is not sensitive to the values of Frisch elasticity.[7]

One the other hand, the value of β is calibrated to generate the interest rate of 3.48% in the steady state under the current Social Security public pension system, while the value of ψ (parameter that captures disutility of working) is selected to beget the associated steady-state employment rate as 64%. Basically, values of all the parameters, including survival rates (m_i), are chosen to yield the values of aggregate US data which are averaged from year 2000 to year 2010. In particular, data on the capital share of output are taken from the US Bureau of Economic Analysis; the capital depreciation rate is

[5] Fuster et. al (2007) adopted 1 as the Frisch elasticity following Chang and Kim (2006). However, Chang and Kim (2006) admitted that their estimate, 1, does not generate enough fluctuation in working hours as real data.

[6] It also is very close to the estimate of Browning, Hansen and Heckman (1999) which is 1.6.

[7] Early estimates such as Altonji (1986) are between 0 and 0.5, which are not consistent with the observed volatility of aggregate labor supply over the business cycle (Chetty et al. 2011). Imai and Keane (2004) extended standard model by incorporating unobservable human capital accumulation and estimated the Frisch elasticity to be 3.85 which is greater than estimates of any other studies such as Cho and Cooley (1994), Chang and Kim (2006), and Chang and Kim (2014).

based on the estimates of Alice Albonico, Sarantis Kalyvitis, and Evi Pappa (2014); and data on the rates of population growth and economic growth and data on the government spending in terms of share of GDP are procured from the World Bank database. According to these data, we calibrate the government expenditure (G) to comprise 15.4% of GDP. Moreover, for ρ and σ_z of the production technology proceeding (8), we adopt the estimates of Komunjer and Ng (2011). The values of parameters calibrated for our simulation analyses are summarized in **Table 1**.

Table 1] Parameters of the Baseline Economy

Capital share of output	α	0.317
Depreciation rate of capital	δ	0.117
Rate of output growth	g	0.019
Population growth rate	n	0.009
Autocorrelation of total factor productivity shock	ρ	0.9
Standard deviation of total factor productivity	σ_z	0.9
Social Security contribution rate	τ_p	0.124
Capital gain tax rate	τ_k	0.25
Time preference (discount factor)	β	0.996
Weight on disutility from work	ψ	5.160
Frisch elasticity of labor supply	η	1.5

For the before-privatization baseline economy, τ_p is set at 12.4% combining the Social Security payroll tax rate of 6.2% paid by employees and by their employers, respectively. Following the Social Security benefit formula, public pension benefit $a_{t,i}$ is calculated as follows.[8]

$$a_{t+1,i+1} = \begin{cases} 0.9\bar{a}_{t,i} \text{ if } \bar{a}_{t,i} \le 0.22\bar{\text{A}} \\ 0.9(0.02\bar{\text{A}})+0.32(\bar{a}_{t,i}-0.02\bar{\text{A}}) \text{ if } 0.22\bar{\text{A}} < \bar{a}_{t,i} \le 1.33\bar{\text{A}} \\ 0.9(0.02\bar{\text{A}})+0.32(0.11\bar{\text{A}}-0.02\bar{\text{A}})+0.15(\bar{a}_{t,i}-0.11\bar{\text{A}}) \text{ if } 1.33\bar{\text{A}} < \bar{a}_{t,i} \le 2.64\bar{\text{A}} \\ 0.9(0.02\bar{\text{A}})+0.32(0.11\bar{\text{A}}-0.02\bar{\text{A}})+0.15(2.47\bar{\text{A}}-0.11\bar{\text{A}}) \text{ if } 2.64\bar{\text{A}} < \bar{a}_{t,i} \end{cases} \quad (22)$$

[8] For further details, visit http://www.ssa.gov/oact/cola/piaformula.html.

if $i < R$, otherwise $a_{t+1,i+1} = a_{t,i}$, where $\bar{\mathrm{A}} \equiv \sum_{i=1}^{I} \int_{E \times K \times A} w_t e l_{t,i}(\mathbf{s}_t) dP_t(\mathbf{s}_t) \{ \sum_{i=1}^{I} \int_{E \times K \times A} dP_t(\mathbf{s}_t) \}^{-1}$ (average labor income of the economy in period t) and $\bar{a}_{t,i} \equiv \frac{1}{i} \sum_{h=1}^{i} w_{t-i+h} e l_{t-i+h,h}$ (average of the past labor incomes of an age-i individual in period t). Individuals become eligible to receive the public pension benefit at the age of 65, so we set R at 45.

As previous studies like Nishiyama and Smetters (2007) did, earning ability e is approximated with data of individuals' annual earnings from the Panel Study of Income Dynamics (PSID) by averaging over the three waves (years of 2003, 2005, and 2007).[9] As described in Section II, individuals of the same age can have different earning abilities, as the earning ability of each individual is subject to idiosyncratic shocks each year. To reflect the ensuing heterogeneity in earning abilities, we first divide each age cohort of the PSID into four income groups with thresholds of $20,000, $40,000, and $60,000; and then we average annual earnings of each of the four income groups for each age cohort. The resulting 4 by 80 matrix is used as earning abilities in our simulation. In addition, to capture the idiosyncratic uncertainty on earning ability of any given age, the probability for an individual to have one of the four levels of earning ability is approximated with the population share of the corresponding income group in the age cohort of the PSID.

IV. SIMULATION RESULTS

A. Effect of Public Pension Privatization on Steady State of Economy

We obtain steady-state equilibria of the pre- and post-privatization economies which solve our model by meeting the conditions from (1) to (16) with the parameters obtained in Section III. The post-privatization economy is different from the pre-privatization baseline economy only in public pension policy by

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[9] Data from those who reported hourly/weekly earnings are converted into annual income based on hours worked.

setting $\tau_p = 0$ and $a_{t,i} = 0$ for $\forall t$ and $\forall i$ for the former, in contrast to $\tau_p = 0.124$ and (22) for the latter. By comparing the two economies that are identical except for the PAYG public pension, we can identify effects of privatizing the PAYG public pension.

Table 2] Steady State Economies Before and After Public Pension Privatization

Pension system	$\bar{L}$	$\bar{K}$	$\bar{Y}$	$\bar{C}$	$\frac{\bar{C}}{\bar{Y}}$	$\bar{r}$	$\bar{w}$	Social welfare
PAYG	1.415 (100.0)	4.159 (100.0)	1.991 (100.0)	1.069 (100.0)	0.537 (100.0)	0.0348 (100.0)	0.961 (100.0)	−0.258 (100.0)
Fully privatized	1.441 (101.8)	4.546 (109.3)	2.074 (104.2)	1.095 (102.4)	0.528 (98.3)	0.0276 (79.4)	0.983 (102.3)	−0.225 (129.3)

Note: The first row reports the stationary equilibrium of the economy under the current Social Security PAYG public pension system, whereas the second row does so for the economy after completion of privatizing the public pension. Numbers in parentheses refer to rescaled values of variables with the before-privatization steady state as 100 in order to express effects of the public pension privatization in percentage terms.

As shown in **Table 2** that summarizes steady-state equilibria before and after the privatization, privatizing PAYG Social Security public pension increases steady-state aggregate capital by 9.3% and aggregate labor supply by 1.8%. Notably, this indicates that the two steady-state conditions for applying **Proposition 1** and **2** are met. The increases in total labor and capital, in turn, raise steady-state total output by 4.2%. As mentioned above, such a pro-growth effect of public pension privatization resonates with other previous studies (e.g., Hubbard and Judd 1987; İmrohoroğlu et al. 2003; Nishiyama and Smetters 2007; Fuster et al. 2007).[10]

As the payroll taxes on labor incomes to finance the Social Security retirement benefits are no longer collected, the public pension privatization removes the labor supply disincentives that the PAYG public pension system imposes. At the same time, since the government ceases to provide public pension insurance, individuals save more for maintaining their post-retirement consumption. This is also consistent with our finding that the public pension

[10] The margin of the increase in steady-state aggregate capital is lower than that of previous studies because of our coefficient of relative risk aversion is lower than theirs.

privatization decreases the share of aggregate consumption in total output ($\frac{\bar{C}}{\bar{Y}}$), although it increases steady-state total consumption, as shown in **Table 2**.

Because the transition process of the public pension privatization is out of the scope of our study and because the steady state equilibrium reached after finishing the privatization is independent of the transition path, we compare steady-state equilibria of the economy before the privatization and the economy after the privatization is completed. The welfare cost from the transition, which varies by assumptions on the transition process, can be subtracted from the steady-state level of social welfare reached after completing the privatization (in the last column of **Table 2**).[11]

B. Effect of Public Pension Privatization on Volatility of Economy and Social Welfare

Having confirmed that privatizing the PAYG Social Security public pension increases steady-state levels of total labor supply and capital, we need to verify whether the privatization decreases variation in total labor supply from responding to an arbitrary given TFP shock, for applying our theoretical findings of **Proposition 1** and **2** to the effect of privatizing Social Security retirement program on the volatilities of economy and social welfare over the business cycle.

To this end, we first simulate series of 1000 TFP shocks by introducing unexpected deviations in the value of ε_t from zero (at the steady state) to randomly generated numbers that take positive and negative values of various magnitudes, according to (8). For each TFP shock, we obtain individuals' post-

[11] Neither in theory nor in practice, there is no clear consensus on the process of the privatization. However, as Huang et al. (1997) and Kotlikoff et al. (1999) showed, overall welfare consequence of public pension privatization is sensitive to welfare cost from the transition process. In fact, the overall welfare gain (or loss) of privatizing PAYG public pensions differs by the transition path. For instance, Nishiyama and Smetters (2007) and Fuster et al. (2007) found that when compensations for during the privatization process are financed by labor income taxes, public pension privatization ends up with a social welfare loss (despite increased labor supply, capital stock, and output), whereas when the compensations are financed by consumption taxes, public pension privatization generates a social welfare gain.

shock responses from solving (17), (18), (19), (20), and (21), which are aggregated with the population weight. Then, we de-trend the growth rates of the post-shock 1000 aggregate variables of labor supply, investment, output, consumption, and social welfare, utilizing Hodrick–Prescott filter with the smoothing parameter of 6.25 following Ravn and Uhlig (2002). To quantify the effect of the public pension privatization on the volatilities of these five macroeconomic variables, we obtain standard deviations of cyclic parts of these de-trended growth rates of aggregate labor supply, investment, output, consumption, and social welfare, whose outcomes are reported in **Table 4**. Before this final stage, we calculate standard deviations of cyclical parts of the de-trended level-change in the five macroeconomic variables (instead of de-trended growth rates of the five aggregate variables) and display the results in **Table 3** to see whether the privatization decreases variation in total labor supply (whether the last remaining condition of **Proposition 1** and **2** is also satisfied or not).

First of all, as shown in **Table 3**, the privatization of Social Security PAYG public pension substantially decreases the variation of total labor supply by 27.3%. Together with the increases in steady-state levels of aggregate labor supply and capital caused by the privatization (**Table 2**), this implies that we can apply **Proposition 1** and **2** to the case of privatizing Social Security public pension. Thus, we can utilize our results for validating and quantifying the theoretical findings of **Proposition 1** and **2**.

Table 3] Variations of Economies Before and After Public Pension Privatization

Public pension	$\lvert\sigma_L\rvert$	$\lvert\sigma_K\rvert$	$\lvert\sigma_Y\rvert$	$\lvert\sigma_C\rvert$	$\lvert\sigma_W\rvert$
PAYG	0.195 (100.0)	6.631 (100.0)	2.156 (100.0)	1.122 (100.0)	0.0107 (100.0)
Fully privatized	0.142 (72.7)	6.539 (98.6)	2.161 (100.2)	1.165 (103.8)	0.0111 (103.7)

Note: $\lvert\sigma_x\rvert$ refers to standard deviation of cyclical parts of level-change in the macroeconomic variable x, which is de-trended with Hodrick–Prescott filter, from its steady state value. The first row reports the variation of the economy under the current PAYG Social Security pension system, whereas the second row does so for the economy after completion of privatizing the public pension. Numbers in parentheses refer to rescaled values of $|\sigma_x|$ with the pre-privatization value as 100 in order to express effects of the public pension privatization in percentage terms.

More importantly, estimated effect of the public pension privatization on the volatilities of macroeconomic variables and social welfare is reported in **Table 4.**[12] We find that privatizing Social Security public pension enhances the stability of total output by 3.8% with decreasing the volatilities of aggregate labor supply and investment by 28.6% and 10.7%, respectively, while the priv atization worsens the instability of social welfare by 18.8% with increasing the volatility of total consumption by 1.3% over the business cycle.

Above all, our findings in **Table 4** are consistent with **Proposition 1** and **2**, as they show that privatizing PAYG public pension decreases the instability of total output by reducing the volatilities of aggregate labor supply and investment, while raising the volatilities of total consumption and social welfare. Essentially, the public pension privatization is found to generate a trade-off between macroeconomic stability and social welfare volatility.

Table 4] Volatilities of Economies Before and After Public Pension Privatization

Public pension	σ_L	σ_K	σ_Y	σ_C	σ_W
PAYG	0.138 (100.0)	1.130 (100.0)	1.083 (100.0)	1.050 (100.0)	0.0414 (100.0)
Fully privatized	0.098 (71.4)	1.009 (89.3)	1.042 (96.2)	1.064 (101.3)	0.0492 (118.8)

Note: σ_x refers to the standard deviation of cyclical parts in the growth rates of the macroeconomic variable x, which is de-trended with Hodrick–Prescott filter. The first row reports the volatility of the economy under the current PAYG Social Security pension system, whereas the second row does so for the economy after completion of privatizing the public pension. Numbers in parentheses refer to rescaled values of σ_x with the pre-privatization value as 100 in order to express effects of the public pension privatization in percentage terms.

Without the PAYG public pension system that could have insured part of individuals' retirement wealth from the macroeconomic risks, individuals now bear the full risks on their retirement wealth, which necessitates their

[12] Our simulation results in **Table 4** maintain the feature of US economy: rank of the volatilities (investment > output> labor supply> consumption).

additional efforts to secure stable provision of retirement consumption. Facing a negative (positive) TFP shock, due to greater loss (gain) in their retirement wealth, individuals reduce (raise) their labor supply and savings with *lesser* degree than before the privatization, while decreasing (increasing) their consumption by *larger* margin. In turn, the resulting decreases in the volatilities of total labor supply and investment (the production inputs) reduce the volatility of total output.

Moreover, as individuals' utilities depend negatively on labor supply and positively on consumption, less reduction (less raise) in labor supply and more reduction (more raise) in consumption, for responding to the negative (positive) TFP shock, cause individuals to suffer (enjoy) more than before the public pension privatization, entailing social welfare to go down (up) further in the post-privatization economy.

V. CONCLUDING REMARKS

This paper investigates the impact of privatizing PAYG public pension on the volatilities of economy and social welfare. First, we theoretically prove that, under some generic and feasible conditions, privatizing PAYG public pension causes total output to be less fluctuating by reducing the volatilities of aggregate labor supply and investment, while increasing the volatilities of social welfare and total consumption over the business cycle.

Second, to estimate such trade-off between macroeconomic and social welfare volatilities which is generated by the public pension privatization, we calibrate the parameters of our model to match the US data and introduce a series of 1000 TFP shocks to the two steady-state economies that are identical except for the PAYG Social Security public pension. We find that the public pension privatization decreases the volatilities of aggregate labor supply and investment by 28.6% and 10.7%, respectively, which reduces the volatility of total output by 3.8% over the business cycle. At the same time, the public pension privatization raises the volatilities of social welfare and total consumption by 18.8% and 1.3%, respectively.

Above all, our study discovers an overlooked benefit of privatizing PAYG public pension — improvement in economic stability by decreasing the volatilities of aggregate labor supply, investment, and output — which turns out not to be given for free but to entail a cost: increase in fluctuations of aggregate consumption and social welfare. Our finding suggests that the government needs to factor in this pair of benefit and cost for evaluating public pension privatization, although how it should weigh the benefit and cost is not within the scope of our study.

REFERENCES

Albonico, Alice, Sarantis Kalyvitis, and Evi Pappa. 2014. "Capital Maintenance and Depreciation over the Business Cycle" *Journal of Economic Dynamics and Control*, 39: 273-286.

Altonji, Joseph G. 1986. "lntertemporal Substitution in Labor Supply: Evidence from Micro Data" *Journal of Political Economy*, 94(3): S176-S215.

Breyer, Friedrich and Martin Straub. 1993. "Welfare Effects of Unfunded Pension Systems When Labor Supply Is Endogenous" *Journal of Public Economics*, 50(1): 77-91.

Browning, M., Hansen, L. P. and Heckman, J. J. 1999. "Micro Data and General Equilibrium Models" In *Handbook of Macroeconomics*, ed. J. B. Taylor and M. Woodford. 543-633. Amsterdam: Elsevier Science.

Chang, Yongsung and Sun-Bin Kim. 2006. "From Individual to Aggregate Labor Supply: A Quantitative Analysis Based on a Heterogeneous Agent Macroeconomy" *International Economic Review*, 47(1): 1-27.

Chang, Yongsung, and Sun-Bin Kim. 2014. "Heterogeneity and Aggregation: Implications for Labor-market Fluctuations: Reply" *American Economic Review*, 104(4): 1461-1466.

Chetty, Raj. 2006. "A New Method of Estimating Risk Aversion" *American Economic Review*, 96(5): 1821-1834.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2011. "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins" *American Economic Review Papers and Proceedings*, 101(3): 471-475.

Cho, Jang-Ok, and Thomas F. Cooley. 1994. "Employment and Hours over the Business Cycle" *Journal of Economic Dynamics and Control*, 18(2): 411-32.

Disney, Richard. 2000. "Crises in Public Pension Programmes in OECD: What Are the Reform Options?" *Economic Journal*, 110: 1-23.

Fiorito, Riccardo and Giulio Zanella. 2012. “The Anatomy of the Aggregate Labor Supply Elasticity” *Review of Economic Dynamics*, 15(2): 171-187.

Fuster, Luisa, Ayse Imrohoroglu, and Selahattin Imrohoroglu. 2007. “Elimination of Social Security in a Dynastic Framework” *Review of Economic Studies*, 74: 113-145.

Huang, H., S. Imrohoroglu, and T. J. Sargent. 1997. “Two Computations to Fund Social Security” *Macroeconomic Dynamics*, 1: 7-44.

Hubbard, R. Glenn and Kenneth L. Judd. 1987. “Social Security and Individual Welfare: Precautionary Saving, Borrowing Constraints, and the Payroll Tax” *American Economic Review*, 77(4): 630-646.

Imai, Susumu and Michael Keane. 2004. “Intertemporal Labor Supply and Human Capital Accumulation” *International Economic Review*, 45(2): 601-641.

İmrohoroğlu, Ayşe, Selahattin İmrohoroğlu, and Douglas Joines. 2003. “Time Inconsistent Preferences and Social Security” *Quarterly Journal of Economics*, 118(2): 745-784.

İmrohoroğlu, Selahattin and Sagiri Kitao. 2009. “Labor Supply Elasticity and Social Security Reform” *Journal of Public Economics*, 93: 867-878.

Komunjer, Ivana and Serena Ng. 2011. “Dynamic Identification of Dynamic Stochastic General Equilibrium Models” *Econometrica*, 79(6): 1995-2032.

Kotlikoff, Laurence. J., Smetters Kent, and Jan Walliser. 1999. “Privatizing Social Security in the US— Comparing the Options” *Review of Economic Dynamics*, 2: 532-574.

Krueger, Dirk and Felix Kubler. 2006. “Pareto-improving Social Security Reform When Financial Markets Are Incomplete!?” *American Economic Review*, 96(3): 737-755.

Nishiyama, Shinichi and Kent Smetters. 2007. “Does Social Security Privatization Produce Efficiency Gains?” *Quarterly Journal of Economics*, 122(4): 1677-1719.

OECD. *Pensions at a Glance 2013: OECD and G20 Indicators*. Paris: OECD Publishing. 2013.

Olovsson, Conny. 2010. “Quantifying the Risk-sharing Welfare Gains of Social Security” *Journal of Monetary Economics*, 57(3): 364-375.

Uhlig, Harald. 1999. “A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily” In *Computational Methods for the Study of Dynamic Economies*, ed. Marimon Scott. 30-61. Oxford: Oxford University Press.

Ravn, O Morten and Harald Uhlig. 2002. “On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations” *Review of Economics and Statistics*, 84(2): 371-380.

APPENDIX

Proof for Proposition 1.

[step 0] For notational convenience, we add superscript *p* to describe the economy after privatizing PAYG public pension. When the privatization of PAYG public pension increases steady-state levels of total labor supply and capital and decreases variation in aggregate labor supply from responding to a TFP shock, $\bar{L}^p - \bar{L} > 0$ and $\bar{K}^p - \bar{K} > 0$; and, for an arbitrarily given TFP shock $\hat{z}_t = \hat{z}_t^p \neq 0$ which equally hits both of pre- and post-privatization economies, $\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p \bar{l}_i^p(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s}) < 0$, since the variation (level-deviation from the steady state value) due to the TFP shock is $L_t - \bar{L} = \sum_{i=1}^{I} \int_{E\times K\times A} e[l_{t,i}(\mathbf{s}) - \bar{l}_i(\mathbf{s})]dP(\mathbf{s}) = \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})\ dP(\mathbf{s})$.

[step 1] By way of contradiction, suppose that the public pension privatization does not decrease the volatility of aggregate labor supply. Then, $\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}(\mathbf{s})dP(\mathbf{s}) \geq 0$. This implies a contradiction to the above assumptions of $\bar{L}^p - \bar{L} = \sum_{i=1}^{I} \int_{E\times K\times A} e\bar{l}_i^{\,p}(\mathbf{s})\ dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} e\bar{l}_i(\mathbf{s})dP(\mathbf{s}) > 0$ and $\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p \bar{l}_i^p(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s}) < 0$. Hence, $\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}(\mathbf{s})dP(\mathbf{s}) < 0$ for an arbitrarily given TFP shock, meaning that the public privatization decreases the volatility of total labor supply when allowing various TPF shocks over the business cycle.

[step 2] We want to show that $\frac{1}{\bar{K}^p}\sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i}^p \bar{k}_i^p(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s}) < 0$ by way of contradiction. So, suppose that $\frac{1}{\bar{K}^p}\sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i}^p \bar{k}_i^p(\mathbf{s})\ dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s}) \geq 0$.

By integrating (18) over the population and comparing the post- and pre-privatization economies, we get $\{\sum_{i=1}^{I} \int_{E\times K\times A} \hat{c}_{t,i}^p dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} \hat{c}_{t,i} dP(\mathbf{s})\} + E[\{\sum_{i=1}^{I} \int_{E\times K\times A} \hat{c}_{t+1,i+1}^p dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} \hat{c}_{t+1,i+1} dP(\mathbf{s})\}] = E[\hat{Y}_{t+1}^p - \hat{Y}_{t+1} - \{\frac{1}{\bar{K}^p}\sum_{i=1}^{I-1} \int_{E\times K\times A} \hat{k}_{t+1,i+1}^p \bar{k}_{i+1}^p$

$(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I-1}\int_{E\times K\times A}\hat{k}_{t+1,i+1}\bar{k}_{i+1}(\mathbf{s})dP(\mathbf{s})\}]$ since $\sum_{i=1}^{I}\int_{E\times K\times A}dP(\mathbf{s}) = 1$. Lagging this equation by one period, we get $\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^{p}_{t-1,i-1}dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t-1,i-1}$ $dP(\mathbf{s})\} + \{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^{p}_{t,i}dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t,i}dP(\mathbf{s})\} = \hat{Y}^{p}_{t} - \hat{Y}_{t} - \{\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^{p}_{t,i}\bar{k}^{p}_{i}(\mathbf{s})$ $dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_{i}(\mathbf{s})dP(\mathbf{s})\}$. Let us this lagged integrated difference of (18) between post- and pre-privatization economies be labeled as (18)′.

Similarly, by integrating (17) over the population and comparing the post- and pre- privatization economies, $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}^{p}_{t,i}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}(\mathbf{s})dP(\mathbf{s}) = \hat{Y}^{p}_{t} -$ $\hat{Y}_{t} - \{\frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}^{p}_{t,i}\bar{l}^{p}_{i}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_{i}(\mathbf{s})dP(\mathbf{s})\} - \frac{1}{\eta}\{\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}^{p}_{t,i}(\mathbf{s})$ $dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}(\mathbf{s})dP(\mathbf{s})\}$, which is labeled as (17)′.

In addition, by integrating (20) over the population and comparing the post- and pre- privatization economies, as $\hat{z}_{t} = \hat{z}^{p}_{t}$, we get $\hat{Y}^{p}_{t} - \hat{Y}_{t} = \alpha\{\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}$ $\int_{E\times K\times A}\hat{k}^{p}_{t,i}\bar{k}^{p}_{i}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_{i}(\mathbf{s})dP(\mathbf{s})\} + (1-\alpha)\{\frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}^{p}_{t,i}\bar{l}^{p}_{i}(\mathbf{s})dP(\mathbf{s})$ $-\frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_{i}(\mathbf{s})dP(\mathbf{s})\}$. Let us label this equation as (20)′.

Combining (17)′ and (20)′, we get $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}^{p}_{t,i}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}(\mathbf{s})dP(\mathbf{s})$ $= \alpha\{\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^{p}_{t,i}\bar{k}^{p}_{i}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_{i}(\mathbf{s})dP(\mathbf{s})\} - \alpha\{\frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}^{p}_{t,i}$ $\bar{l}^{p}_{i}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_{i}(\mathbf{s})dP(\mathbf{s})\} - \frac{1}{\eta}\{\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}^{p}_{t,i}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}(\mathbf{s})$ $dP(\mathbf{s})\}$. Firstly, as supposed at the beginning of [step 2], the first term is positive, since $\alpha > 0$. Secondly, due to [step 1], the second term is also positive. Thirdly, the third term is positive also due to the [step 1]. Taking these together, $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}^{p}_{t,i}(\mathbf{s})dP(\mathbf{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}(\mathbf{s})dP(\mathbf{s}) > 0$. This implies that the left hand side of (18)′ $\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^{p}_{t-1,i-1}dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t-1,i-1}dP(\mathbf{s})\} + \{$

$\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^p_{t,i}dP(\mathbf{s})-\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t,i}dP(\mathbf{s})\}>0$ as $\hat{c}^p_{t-1,0}=\hat{c}_{t-1,0}=0=\hat{c}^p_{t-1,I+1}=\hat{c}_{t-1,I+1}$ (no one alive is of age $I+1$ and age *i* starts from 1 in our model).

Then, by combining (18)′ and (20)′, we get $\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^p_{t-1,i-1}dP(\mathbf{s})-\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t-1,i-1}dP(\mathbf{s})\}+\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^p_{t,i}dP(\mathbf{s})-\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t,i}dP(\mathbf{s})\}=(\alpha-1)\{\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^p_{t,i}\bar{k}^p_i(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s})\}+(1-\alpha)\{\frac{1}{\bar{L}^p}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}^p_{t,i}\bar{l}^p_i(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s})\}$. As we just show that the left-hand side of this equation is positive, the sign of the right-hand side should be positive as well. Moreover, the second term of the right-hand side is negative due to [step 1] and $1>\alpha>0$, which implies that the first term should be positive. This is a contradiction to the above assumption at the beginning of this step, which proves that $\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^p_{t,i}\bar{k}^p_i(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s})<0$. Since $\bar{K}^p-\bar{K}=\sum_{i=1}^{I}\int_{E\times K\times A}\bar{k}^p_{t,i}(\mathbf{s})dP(\mathbf{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\bar{k}_{t,i}(\mathbf{s})dP(\mathbf{s})>0$, this implies that $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^p_{t,i}(\mathbf{s})dP(\mathbf{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}(\mathbf{s})dP(\mathbf{s})<0$ for an arbitrarily given TFP shock. Therefore, the privatization decreases the volatility of aggregate investment over the business cycle.

[step 3] From [step 1] and [step 2], we prove that $\{\frac{1}{\bar{K}^p}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^p_{t,i}\bar{k}^p_i(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s})\}<0$ and $\frac{1}{\bar{L}^p}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}^p_{t,i}\bar{l}^p_i(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s})<0$. This implies that $\hat{Y}^p_t-\hat{Y}_t<0$ for an arbitrarily given TFP shock, due to (20)′ and $1>\alpha>0$, which means that the public pension privatization decreases the volatility of total output over the business cycle. **Q.E.D**

Proof for Proposition 2.

[step 0] At the outset, let us explicitly state the volatility of social welfare. Since individuals' utilities are at their maximums at steady state equilibrium, before any TFP shock hits, social welfare is $\sum_{i=1}^{I}\int_{E\times K\times A}V_i(\mathbf{s})dP(\mathbf{s})=\sum_{i=1}^{I}\int_{E\times K\times A}$

$u(\overline{c}_i, \overline{l}_i) + m_i \beta E[V_{i+1}(\mathbf{s})] dP(\mathbf{s})$. When a TFP shock hits, utility responses of each individual are realized via deviations of their *current* labor supply and consumption from their own steady-state levels. Therefore, the entailed change in their utility brought by the TFP shock is $u(c_{t,i}, l_{t,i}) + m_i \beta E[V_{i+1}(\mathbf{s})] - \{$ $u(\overline{c}_i, \overline{l}_i) + m_i \beta E[V_{i+1}(\mathbf{s})]\} = u(c_i, l_i) - u(\overline{c}_i, \overline{l}_i)$. So, the entailed level-change in social welfare is $\sum_{i=1}^{I} \int_{E\times K\times A} [u(c_{t,i}, l_{t,i}) - u(\overline{c}_i, \overline{l}_i)] dP(\mathbf{s})$, which takes the opposite sign if it is divided by a negative $\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\mathbf{s}) dP(\mathbf{s})$. Thus, the volatility of social welfare is $s\hat{w}_t = sign(\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\mathbf{s}) dP(\mathbf{s})) \sum_{i=1}^{I} \int_{E\times K\times A} \hat{v}_{t,i} dP(\mathbf{s})$ where $\hat{v}_{t,i} =$ $\dfrac{u(c_{t,i}, l_{t,i}) - u(\overline{c}_i, \overline{l}_i)}{\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\mathbf{s}) dP(\mathbf{s})}$; and $sign(x)$ is a function that returns the sign of x.

[step 1] For notational convenience, we add superscript p to describe the economy after privatizing PAYG public pension. When the privatization of PAYG public pension increases steady-state levels of total labor supply and capital and decreases variation in total labor supply from responding to a TFP shock, $\overline{L}^p - \overline{L} > 0$ and $\overline{K}^p - \overline{K} > 0$; and, for an arbitrarily given TFP shock $\hat{z}_t = \hat{z}_t^p \neq 0$ which equally hits both of pre- and post-privatization economies, $\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p \bar{l}_i^p(\mathbf{s}) dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i} \bar{l}_i(\mathbf{s}) dP(\mathbf{s}) < 0$. According to the step 2 of proof for **Proposition 1**, this implies (i) $\{\frac{1}{\overline{K}^p} \sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i}^p \overline{k}_i^p(\mathbf{s}) dP(\mathbf{s}) - \frac{1}{\overline{K}} \sum_{i=1}^{I}$ $\int_{E\times K\times A} \hat{k}_{t,i} \overline{k}_i(\mathbf{s}) dP(\mathbf{s})\} < 0$, (ii) $\frac{1}{\overline{L}^p} \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p \bar{l}_i^p(\mathbf{s}) dP(\mathbf{s}) - \frac{1}{\overline{L}} \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i} \bar{l}_i(\mathbf{s})$ $dP(\mathbf{s}) < 0$, and (iii) $\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p(\mathbf{s}) dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}(\mathbf{s}) dP(\mathbf{s}) < 0$. First of all, as shown in the step 3 of proof for **Proposition 1**, (i) and (ii) imply that the public pension privatization decreases the volatility of total output.

[step 2] We want to show that $\sum_{i=1}^{I} \int_{E\times K\times A} \hat{c}_{t,i}^p(\mathbf{s}) dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} \hat{c}_{t,i}(\mathbf{s}) dP(\mathbf{s}) > 0$ by way of contradiction. So, suppose that $\sum_{i=1}^{I} \int_{E\times K\times A} \hat{c}_{t,i}^p(\mathbf{s}) dP(\mathbf{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} \hat{c}_{t,i}$ $(\mathbf{s}) dP(\mathbf{s}) \leq 0$. Combining $(17)'$ and $(20)'$ in the step 2 of proof for **Proposition**

1, we obtain $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}(\mathbf{s})dP(\mathbf{s})=\alpha[\{\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\bar{k}_{i}^{p}$ $(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_{i}(\mathbf{s})dP(\mathbf{s})\}-\{\frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}\bar{l}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e$ $\hat{l}_{t,i}\bar{l}_{i}(\mathbf{s})dP(\mathbf{s})\}]-\frac{1}{\eta}\{\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s})-\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}(\mathbf{s})dP(\mathbf{s})\}$. This implies that $[\{\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\bar{k}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_{i}(\mathbf{s})dP(\mathbf{s})\}-\{\frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}$ $\bar{l}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_{i}(\mathbf{s})dP(\mathbf{s})\}]<0$. On the other hand, combining (18)′ and (20)′ in the step 2 of proof for **Proposition 1**, we get $\{\sum_{i=1}^{I+1}$ $\int_{E\times K\times A}\hat{c}_{t-1,i-1}^{p}dP(\mathbf{s})-\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t-1,i-1}dP(\mathbf{s})\}+\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t,i}^{p}dP(\mathbf{s})-\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{t,i}dP(\mathbf{s})\}=$ $(\alpha-1)[\{\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\bar{k}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_{i}(\mathbf{s})dP(\mathbf{s})\}-\{\frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}$ $\bar{l}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_{i}(\mathbf{s})dP(\mathbf{s})\}]>0$; A contradiction to the assumption at the beginning of this step. So, $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}^{p}(\mathbf{s})dP(\mathbf{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{t,i}(\mathbf{s})dP(\mathbf{s})$ >0 which implies that the public pension privatization increases the volatility of aggregate consumption.

[step 3] We want to show that $\hat{c}_{t,i}^{p}-\hat{c}_{t,i}>0$ for any given *i*. By comparing (18) of the post- and pre- privatization economies, we get $\hat{c}_{t,i}^{p}-\hat{c}_{t,i}+\hat{c}_{t+1,i+1}^{p}-\hat{c}_{t+1,i+1}$ $=\hat{Y}_{t}^{p}-\hat{Y}_{t}-\{\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\bar{k}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_{i}(\mathbf{s})dP(\mathbf{s})\}$. Combining this equation with (20)′ of the step 2 of proof for **Proposition 1**, $\hat{c}_{t,i}^{p}$ $-\hat{c}_{t,i}+\hat{c}_{t+1,i+1}^{p}-\hat{c}_{t+1,i+1}=(\alpha-1)[\{\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}^{p}\bar{k}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{t,i}\bar{k}_{i}(\mathbf{s})dP$ $(\mathbf{s})\}-\{\frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}^{p}\bar{l}_{i}^{p}(\mathbf{s})dP(\mathbf{s})-\frac{1}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{t,i}\bar{l}_{i}(\mathbf{s})dP(\mathbf{s})\}]>0$ since [step 2] above shows that the right-hand side of this equation is positive, which implies that $\hat{c}_{t,i}^{p}-\hat{c}_{t,i}>0$.

[step 4] We examine whether the increase in individual consumption volatility, which is shown in [step 3] above, raises the volatility of social welfare or not,

which can be shown by the sign of $\frac{\mathrm{d}s\hat{w}_t}{\mathrm{d}\hat{c}_{t,i}}$. Notice that $\frac{\mathrm{d}s\hat{w}_t}{\mathrm{d}\hat{c}_{t,i}} = \frac{ds\hat{w}_t}{d\hat{v}_{t,i}} \frac{d\hat{v}_{t,i}}{dc_{t,i}} \frac{dc_{t,i}}{d\hat{c}_{t,i}} = \frac{ds\hat{w}_t}{d\hat{v}_{t,i}}[\frac{d\hat{v}_{t,i}}{dc_{t,i}} \frac{dc_{t,i}}{d\hat{c}_{t,i}}]$. Firstly, $\frac{d\hat{v}_{t,i}}{dc_{t,i}} \frac{dc_{t,i}}{d\hat{c}_{t,i}} = \frac{u_c(\bar{c}_i \exp(\hat{c}_{t,i}))}{\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})}$, as $c_{t,i} = \bar{c}_i \exp(\hat{c}_{t,i})$. Since $u_c > 0$, the sign of $\frac{d\hat{v}_{t,i}}{dc_{t,i}} \frac{dc_{t,i}}{d\hat{c}_{t,i}}$ is equal to $sign(\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\mathbf{s}) dP(\mathbf{s}))$. Secondly, the sign of $\frac{ds\hat{w}_t}{d\hat{v}_{t,i}}$ is equal to $sign(\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))$. Taken together, $sign(\frac{\mathrm{d}s\hat{w}_t}{\mathrm{d}\hat{c}_{t,i}}) = sign(\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))sign(\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})) > 0$, which implies that the public pension privatization raises the volatility of social welfare (i.e., $s\hat{w}_t^p - s\hat{w}_t > 0$) via the increase in the consumption volatility from [step 3].

[step 5] We want to show that $e\hat{l}_{t,i}^p - e\hat{l}_{t,i} < 0$ for any given *i*. By comparing (17) of the post- and pre- privatization economies, we get $\frac{1}{\eta}\{e\hat{l}_{t,i}^p - e\hat{l}_{t,i}\} = \hat{Y}_t^p - \hat{Y}_t - \{\frac{1}{\bar{L}^p}\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p \bar{l}_i^p(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{L}}\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s})\} - \{\hat{c}_{t,i}^p - \hat{c}_{t,i}\}$.

Combining this equation with (20)′ of the step 2 of proof for **Proposition 1**, we get $\frac{1}{\eta}\{e\hat{l}_{t,i}^p - e\hat{l}_{t,i}\} = \alpha[\{\frac{1}{\bar{K}^p}\sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i}^p \bar{k}_i^p(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i}\bar{k}_i(\mathbf{s})dP(\mathbf{s})\} - \{\frac{1}{\bar{L}^p}\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p \bar{l}_i^p(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{L}}\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s})\}] - \{\hat{c}_{t,i}^p - \hat{c}_{t,i}\}$. As shown in [step 2] above, $[\{\frac{1}{\bar{K}^p}\sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i}^p \bar{k}_i^p(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}}\sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{t,i}\bar{k}_i(\mathbf{s}) dP(\mathbf{s})\} - \{\frac{1}{\bar{L}^p}\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}^p \bar{l}_i^p(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{L}}\sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{t,i}\bar{l}_i(\mathbf{s})dP(\mathbf{s})\}] < 0$, so the first term is negative; and due to [step 3] above, the last term ($-\{\hat{c}_{t,i}^p - \hat{c}_{t,i}\}$) is also negative. Therefore, $e\hat{l}_{t,i}^p - e\hat{l}_{t,i} < 0$.

[step 6] Next, we examine whether the decrease in individual labor supply volatility, which is shown in [step 5] above, raises social welfare volatility or not by finding the sign of $-\frac{\mathrm{d}s\hat{w}_t}{\mathrm{d}\hat{l}_{t,i}}$. Notice that $\frac{\mathrm{d}s\hat{w}_t}{\mathrm{d}\hat{l}_{t,i}} = \frac{ds\hat{w}_t}{d\hat{v}_{t,i}}[\frac{d\hat{v}_{t,i}}{dl_{t,i}} \frac{dl_{t,i}}{d\hat{l}_{t,i}}]$. Firstly,

$\frac{d\hat{v}_{t,i}}{dl_{t,i}}\frac{dl_{t,i}}{d\hat{l}_{t,i}} = \frac{u_l(\bar{l}_i \exp(\hat{l}_{t,i}))}{[\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})]}$. Because $u_l < 0$, the sign of $\frac{d\hat{v}_{t,i}}{dl_{t,i}}\frac{dl_{t,i}}{d\hat{l}_{t,i}}$ is equal to $-sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))$. Secondly, the sign of $\frac{ds\hat{w}_t}{d\hat{v}_{t,i}}$ is equal to $sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))$. Thus, $sign(-\frac{\mathrm{d}s\hat{w}_t}{\mathrm{d}\hat{l}_{t,i}}) = sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s}))sign(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\mathbf{s})dP(\mathbf{s})) > 0$. Therefore, the public pension privatization raises the volatility of social welfare (i.e., $s\hat{w}_t^p - s\hat{w}_t > 0$) via the increase in the labor supply volatility from [step 5].

[step 7] Finally, taking the above six steps together implies that the public pension privatization increases the volatility of social welfare, while decreasing the volatility of total output and increasing the volatility of aggregate consumption over the business cycle. **Q.E.D**