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P H B S

Bequest and Moral Hazard in Family

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I. Introduction

Bequest behavior is important in economics.[1] Nonetheless, we do not have a model that can coherently explain observed bequest behavior across different societies. First, inheritance rules in myriads of vastly diverse societies across different times and regions are highly polarized into unigeniture (giving all to one child) and equigeniture (dividing bequests equally). For example, 91.9 % of 350 pre-industrial societies around the world adopted one of these two extreme rules of inheritance (Murdock 1967).[2] Second, once inheritance rule of a society evolves from unigeniture to equigeniture, it barely reverted. At the end of this evolutionary trend, in industrialized countries, parents practice equigeniture.[3] This set of observations is neither trivial nor consistent with existing theories on bequest behavior. As choice environment that parents (testators) face differs enormously, it is not immediately explicable that the chosen inheritance rules are not spread out over other various non-extreme forms but concentrated on the two extremes of unigeniture and equigeniture. This paper aims to present a theoretical model that can coherently rationalize the observed bequest behavior across societies.

To date, there are numerous studies on bequest behavior; nevertheless, none directly addressed the aforementioned observations on bequest behavior across societies. Among various models on bequest motive, the following two models drew most scholars' attention and competed each other.[4] First, altruism model,

[1] For instance, bequest motive is crucial not only for Ricardian equivalence but also for behavioral responses to policies to curve inequality evolving over generations.

[2] This polarization of inheritance rules is also found in data from the *Encyclopedia of World Cultures* (Levinson 1991).

[3] Many studies with various datasets of the US find that parents divide bequests equally among their children (e.g., Menchik 1980; Dunn and Phillips 1997; Behrman and Rosenzweig 2004, etc.). Moreover, Kohli (2004) reports that equigeniture is also observed in France, Germany, Israel, Norway, and Sweden.

[4] A brief summary of the various models on bequest motive is provided in Kopczuk (2009).

proposed by Becker (1974), postulates that a parent embraces his children' utilities into his own utility and always transfers resources (bequests) to each child. Second, exchange motive model, proposed by Bernheim et al. (1985), states that a parent strategically utilizes bequests to induce affective attentions from his children. However, none of these two main models can explain the prevalence of equigeniture or unigeniture in a society; in these models, equigeniture is an exceptional 'knife-edge' case ("equal division puzzle") and unigeniture is not a supported solution.

More closely related to the topic of this paper, few papers analyzed equigeniture and/or unigeniture. Chu (1991) and DeLong (2003) argued that primogeniture (a type of unigeniture) is chosen in order to raise social class of a family lineage via accumulation of wealth over generations. However, their argument cannot explain adoptions of equigeniture in many pre-industrial non-egalitarian societies such as medieval China or India where social class of an individual was determined by social class of a family he was born into; in their models, equigeniture is not optimal as it lowers social class of family lineage. Moreover, Bernheim and Severinov (2003) elaborated on the conditions that give rise to equigeniture and unigeniture, respectively, maintaining that bequest works as a signal for parent's preference which each child wants but does not know. In particular, they argue that a society with higher social mobility is more likely to adopt equigeniture as its inheritance rule than unigeniture. Similarly, Lundholm and Ohlsson (2000) maintained that an egalitarian social norm leads parents to adopt equigeniture; however, their assumption that equality is desired by parents is vital for obtaining equigeniture as equilibrium, which leaves their argument tautological. Above all, Bernheim and Severinov (2003) and Lundholm and Ohlsson (2000) are inconsistent with such observations that equigeniture was already adopted in societies with little social mobility and non-egalitarian social norms such as medieval China or India under strict caste system, while primogeniture was practiced in England until 1926. Markedly, the observed polarization in inheritance rules of immensely different societies is not explained by any of the previous studies.

This paper highlights that a parent leads his family and pursues to well perform the main function of family. As the leader of a family team, a parent not only cares about welfare of family members (his children) but also wants his children to exert efforts for performing the main function of family. Thus, the parent needs to induce costly efforts from his children by rewarding them with bequests. However, individual efforts of each child are not verifiable to a third party who enforces the will outside the family. This information asymmetry between inside and outside family generates moral hazard where children do not expend as much efforts as their parent desires. From our model, we show that all the stable equilibrium inheritance rules consist only of unigeniture and equigeniture. Moreover, we find that a rise in the productivity of efforts for family causes optimal inheritance rule to evolve from unigeniture to equigeniture. This implies that non-egalitarian pre-industrial agrarian societies, where the main function of family is subsistence, like feudal China or India, choose equigeniture over unigeniture due to their high agricultural labor productivity. Industrialization, which changed the main function of family from subsistence to emotional support, entailed equigeniture, since the resulting transformation of physical effort to psychological one raised the productivity of efforts for family.

This paper is organized as follows. Section II presents a theoretical model of bequest behavior. Section III characterizes all the stable equilibria, whose evolution is analyzed in Section IV. Finally, Section V concludes the paper.

II. Model

As one of the oldest and most ubiquitous organizations, family has long served various needs of human beings. For most of human history, the main function of family has been subsistence: Family members worked together, as a team, to produce food for their family to subsist. However, as industrialization progressed, better income sources outside a family became increasingly available to the family members; as a result, team production for economic survival has become a less principal task to family. Over industrialization, many roles that family had traditionally performed have been commercialized or taken over by institutions outside family. In the end, emotional betterment — special deep bonding, prestige and esteem (Becker 1981), and the like — has become the primary function of family, as no close substitute is available outside a family.

Throughout such change in the main function of family, a parent has long remained in charge of leading his family and has pursued to fulfill its main function, which needs inputs of efforts from his children (family members). To describe this, consider a family production function, $F(a_1,\cdots,a_n):\Re_+^n \to \Re_+$, where children are indexed by $i \in \{1,\cdots,n\}$ and a_i is effort of child *i*. Suppose that $F(\mathbf{a})$ is continuously differentiable, symmetric[5] with $F(0,\cdots,0)=0$, strictly increasing in each argument ($F_i > 0$ for $\forall i$) and follows the law of diminishing marginal returns ($F_{ii} < 0$ for $\forall i$). Moreover, the family production is sum of contributions from individual children who are eligible inheritors[6]; that is, let $F(a_1,\cdots,a_n)=\sum_{i=1}^{n} f(a_i)$. This implies that $f' > 0$ and $f'' < 0$. Because we are analyzing *distribution* of bequests among children, the only-child case is not of interest; hence, let $n > 1$. Notably, as the main function of family evolved from economic subsistence to emotional support after industrialization, the output of

[5] Symmetricity means that one unit of effort is the same regardless of which child provides it, as it is the same input for family production.

[6] In this study that analyzes the *distribution* of bequests among the eligible inheritors, note that kinship system (such as patrilineal or matrilineal kinship) that determines who are eligible inheritors of a family is exogenously given.

$F(a_1, \cdots, a_n) = F(\mathbf{a}) \equiv q$ changed accordingly. That is, when the primary function of family is economic survival, the output of $F(\mathbf{a})$ refers to produced crops; and, when the main function of family is psychological support, the output refers to emotional betterment of family. In this line, the input for $F(\mathbf{a})$ is accordingly defined as manual effort in the former case and as emotional effort in the latter case.

As a leader of family, the parent pursues greater output of the family production q; so, he wants his children to spend more efforts for family. However, as efforts cost time and energy, the parent compensates his children for their efforts to induce their effort for the family production. At the same time, the parent also cares about happiness of his children so that he internalizes their utilities into his own utility. Taken together, the utility of the parent U_p is stated as

(1) $$U_p = \sum_{i=1}^{n} f(a_i) - \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} (U_i + \mu_i),$$

where t_i is a material reward for child *i*'s effort $a_i \in \Re_+$; U_i is child *i*'s utility; and, μ_i is personal value that the parent attaches specifically to child *i*. To finance the rewards to each of his children from the total output of family team production $\sum_{i=1}^{n} f(a_i) = q$, the total output, either stacks of crops or emotional betterment, is transformed into materials transferable to his children (such as land) after their efforts are provided. Furthermore, let child *i*'s utility U_i be defined as

(2) $$U_i = t_i - c(a_i)$$

where $c(a_i)$ is a cost function. $c(a_i)$ is continuously differentiable, increasing ($c' \geq 0$), and convex ($c'' \geq 0$) with $c(0) = c'(0) = 0$.

Although the parent's utility depends on his children's efforts $\mathbf{a} = (a_1, \cdots, a_n)$, the parent cannot directly choose any of a_i, which is chosen by child *i* himself. Nevertheless, the parent can induce effort a_i from child *i* by offering a material reward t_i to him, based on (2). Thus, the parent chooses a reward scheme $\mathbf{t} = (t_1, \cdots, t_n)$ to induce $\mathbf{a} = (a_1, \cdots, a_n)$ that maximizes his payoff U_p. The time line is as follows: (i) the parent announces a transfer scheme of $(t_1, \cdots, t_n)$; (ii) each child expends his own effort a_i while the parent is alive; and, (iii) the transfer scheme is executed after the death of the parent.[7]

The parent wants more efforts from his children, while his children do not want to spend costly efforts. Thus, the preferences of the two sides are not aligned but conflicted. Hence, any transfer scheme of $(t_1, \cdots, t_n)$ is not self-enforceable; and, any disputes on a transfer scheme in the implementation stage (iii) cannot be resolved impartially inside the family. Without effective enforcement, the parent cannot make a credible commitment to a reward scheme that he announces in (i), which disables the parent from inducing his children's efforts in (ii). For a transfer scheme to be enforceable so that the parent's commitment to his announced transfer scheme in (i) is credible to each child, there should be an impartial third party who executes a transfer scheme in (iii) and resolves any disputes over the transfer scheme. To be impartial, such a third party should be outside the family — for example, a judge at probate court. Thus, our model assumes existence of such an enforcer third party outside family so that the parent can credibly commit to a transfer scheme announced at (i) and each child *i* can decide a_i based on the announced reward scheme t_i by maximizing his own utility (2) in (ii).

Moreover, the parent will make a transfer scheme $(t_1, \cdots, t_n)$ a *contingent* plan, because promising constant rewards at (i) induces zero effort from his children.

[7] To efficiently analyze inheritance rules (norms) in a wide variety of societies, we simplify the time line into three steps without revisions of the transfer scheme, or will in modern western societies, before the parent dies. In this line, for the present analysis, we focus on post-mortem transfer, bequest. Extending our model to include other forms of transfers like inter-vivos transfers or to include revision of wills is for a future study.

To make a contingent reward scheme announced at (i) credibly enforceable, such a reward scheme must be contingent on the information that is verifiable to an enforcer third party (Hart 1987). Because the third party enforcer is outside family, individual effort of each child is not observable to the enforcer. The parent is not alive for verification in (iii). In the team work of producing crops together inside the family, individual effort of each child is not separately verifiable to a third party outside the family. When the main function of family is emotional support, psychological effort of each individual child is not observable to anybody; so, it is not verifiable at all. Therefore, an enforceable transfer scheme cannot be contingent on $(a_1, \cdots, a_n)$.

As an enforceable transfer scheme cannot hinge directly on a_i, we need an alternative contingency that is verifiable *and* positively associated with individual efforts of each child. To this end, we consider the final output of family production q which is transformed into transferable materials such as land, monetary assets, and the like. Such a bequeathable material form of q is observable by and thus verifiable to a third party outside the family; moreover, it depends positively on children's efforts. Therefore, the final output q can serve as a contingency for an enforceable transfer scheme. Thus, an enforceable transfer scheme $\mathbf{t}$ announced by the parent in (i) is contingent on the total output q; that is, $\mathbf{t} = (t_1(q), \cdots, t_n(q))$, a vector-valued function of final output.

Moreover, it is unreasonable for the parent to select a transfer scheme $(t_1(q), \cdots, t_n(q))$ that is decreasing in q, because when more output entails less reward, spending efforts will be discouraged, as opposed to what the parent wants. Thus, we impose the following no-discouragement condition

(3) $$\frac{dt_i}{dq} \geq 0 \text{ for } \forall i .$$

In addition, because the sum of reward payments cannot exceed the final output and it is neither efficient nor credible for the parent to leave the final output unused or to discard the final output after he dies,

(4) $$\sum_{i=1}^{n} t_i(q) = q \text{ for } \forall q \in \Re_+$$

For inducing $\mathbf{a} = (a_1, \cdots, a_n)$, the parent should make an enforceable transfer scheme $(t_1(q), \cdots, t_n(q))$ compatible with his children's incentives to provide $\mathbf{a}$; otherwise, the transfer scheme is not feasible to implement effort levels that the parent intends to induce. Namely, $\mathbf{a}$ is implementable if and only if there exists an enforceable transfer scheme $\mathbf{t}$ such that

(5) $$U_i(t_i(F(\mathbf{a})), a_i) \geq U_i(t_i(F(a_i'; \mathbf{a}_{-i})), a_i') \text{ for } \forall a_i' \in \Re_+ \text{ and } \forall i$$

where $(a_i'; \mathbf{a}_{-i})$ is a vector of efforts which is identical to $\mathbf{a}$ except for the ith element a_i'.

As an enforceable transfer scheme $\mathbf{t}$ is a vector-valued function, the parent's solving for optimal $\mathbf{t}$ involves finding a *function*, which is much more complex and less tractable than finding values of variables. Let an enforceable transfer scheme $\mathbf{t}$ be piecewise continuously differentiable to allow for second-order approximations of $\mathbf{t}$. Then, finding an optimal transfer scheme $\mathbf{t}$ can be streamlined by the following lemma.

Lemma 1. For any given implementable $\mathbf{a}$ with an enforceable transfer scheme $\mathbf{t}$, a parent can always find an affine transfer scheme that induces exactly the same efforts $\mathbf{a}$ from his children as the transfer scheme $\mathbf{t}$ does.

Proof. First, we want to show that $\mathbf{t}$ is continuous in $\mathbf{a}$. Suppose not; that is, there exists $\delta > 0$ such that $t_i(F(\mathbf{a}) + \varepsilon) - t_i(F(\mathbf{a}) - \varepsilon) \geq \delta$ for some i and $\varepsilon > 0$. Then, pick any $\varepsilon \in (0, \frac{\delta}{2})$. Due to the resource constraint (4), $\sum_{i=1}^{n} t_i(F(\mathbf{a}) + \varepsilon)$

$-\sum_{i=1}^{n} t_i(F(\mathbf{a})-\varepsilon) = F(\mathbf{a})+\varepsilon-(F(\mathbf{a})-\varepsilon) = 2\varepsilon \geq \delta$. This is a contradiction to $\varepsilon \in (0, \frac{\delta}{2})$. Therefore, $\mathbf{t}$ is continuous in $\mathbf{a}$.

Because $\mathbf{t}$ is piecewise continuously differentiable, the continuity (shown right above) implies that there always exists and thus a parent can always find $\eta > 0$ such that, in an open set of $(F(\mathbf{a})-\eta, F(\mathbf{a})+\eta)$, the first-order condition of maximizing U_i is met; that is, $t_i'(F(\mathbf{a}))f' - c'(a_i) = 0$ for $\forall i$ because $\mathbf{t}$ is implementable, meeting the incentive compatibility condition (5), and because $c(a_i)$ is continuously differentiable. Let's denote $b_i \equiv \frac{c'(a_i)}{f'} = t_i'$ for $\forall i$. Because $f' > 0$, b_i is always defined. Moreover, as $c' \geq 0$, b_i is non-negative for $\forall i$, meeting the no-discouragement condition (3) as well. In addition, let $g_i \equiv t_i - b_i F(\mathbf{a})$ for $\forall i$. Now, consider a vector-valued function $\hat{\mathbf{t}}$ such that $\hat{t}_i = g_i + b_i F(\mathbf{a}) = g_i + b_i q$ for $\forall i$. By construction, $\hat{t}_i = t_i$ for $\forall i$, and $\sum_{i=1}^{n} \hat{t}_i = \sum_{i=1}^{n} t_i = q$ satisfying (4). Most of all, this affine transfer scheme $\hat{\mathbf{t}}$ induces exactly the same effort levels $\mathbf{a}$ that the original $\mathbf{t}$ will implement because $\hat{t}_i' F(\mathbf{a}) f' - c'(a_i) = t_i' F(\mathbf{a}) f' - c'(a_i) = 0$ for $\forall i$. ■

Notice that **Lemma 1** simplifies the parent's problem of finding a function into finding values of n pairs of (b_i, g_i) since $t_i(q) = g_i + b_i F(\mathbf{a}) = g_i + b_i q$ for $\forall i$. Moreover, $\mathbf{b} = (b_1, \cdots, b_n)$ lies in an n-dimensional simplex Δ^n because

(6) $$\sum_{i=1}^{n} t_i' = \sum_{i=1}^{n} b_i = 1 \text{ and } b_i \geq 0 \text{ for } \forall i$$

based on (3), (4), and the proof of **Lemma 1**.

To find an optimal transfer scheme $\mathbf{t}$ inducing children's efforts that maximize U_p, the parent should first know how his children would respond to an announced transfer scheme. In (ii), given an enforceable reward scheme $\mathbf{t}$, i.e., n pairs of (b_i, g_i), each child i will choose his effort a_i by maximizing U_i; thus,

(7) $$a_i = \arg\max_{a_i \in \Re_+} g_i + b_i F(a_i;\mathbf{a}_{-i}) - c(a_i)$$

Because $F_i(a_i;\mathbf{a}_{-i}) = \dfrac{\partial F(a_i;\mathbf{a}_{-i})}{\partial a_i} = f'(a_i)$, the first-order necessary condition (henceforth, FOC) of (7) is

(8) $$b_i f'(a_i) - c'(a_i) = 0$$

which is also sufficient due to the convexity of c, the concavity of f, and (6). In other words, (8) is equivalent to the incentive compatibility constraints (IC constraints) of (5) for any given i. Furthermore, as $f' > 0$, we can obtain a well-defined function γ such that

(9) $$\gamma(a_i) \equiv \frac{c'(a_i)}{f'} = b_i$$

In line with the no-discouragement condition (3), we can exclude a decreasing γ because if more effort yields less reward, the children would stop exerting effort. Therefore, we focus on a function γ that is strictly increasing in a_i for $\forall i$; that is, let

(10) $$\gamma' > 0 .$$

Then, we can derive an inverse function of γ as

(11) $$a_i(b_i) \equiv \gamma^{-1}(b_i) .$$

which is the best response function of child i. Taken together, the parent's problem of finding an optimal transfer scheme $\mathbf{t}$ can be stated as

(12) $$\max_{\mathbf{b}\in\Delta^n} \sum_{i=1}^n f(a_i(b_i)) - \sum_{i=1}^n c(a_i(b_i)) \text{ s.t. } b_i f'(a_i) - c'(a_i) = 0 \text{ for } \forall i .$$

Notice that μ_is and g_is are omitted since these do not affect maximizations. That is, μ_is, as constant terms, do not affect the parent's maximization; and, as appears in (8), g_is do not affect his children's decisions of their efforts. Therefore, it is sufficient to find an optimal inheritance rule $\mathbf{b}^* = (b_1^*, \cdots, b_n^*)$ that solves (12). In detail, the g_i s can freely take any values as long as $\sum_{i=1}^{n} g_i = 0$ due to (4) and (6). Nevertheless, because the parent treats the equal amount of effort from each child equally, he may as well set the value of the g_i s as

(13) $$g_i = 0 \text{ for } \forall i .$$

Above all, it is worthwhile to notice that the parent's problem (12) is essentially moral hazard problem in his family team (Holmstrom 1982). The lack of verifiability of individual efforts of the parent's children creates moral hazards so that the parent's children do not exert efforts as much as the parent wants. In other words, the parent cannot induce each of his children to expend first-best level of efforts. The first-best outcome is obtained when the parent can observe individual efforts of his children and impartially enforce his transfer scheme **t** at the implementation stage (iii) in a credible way so that the reward scheme **t** announced at (i) can be contingent on $(a_1, \cdots, a_n)$ obviating the need to impose the incentive constraints (5), or equivalently (8), in maximizing U_p. The parent can achieve to make each of his children choose the first-best level of effort $\mathbf{a}^{FB} = (a_1^{FB}, \cdots, a_n^{FB})$ which is defined by $f'(a_i^{FB}) - c'(a_i^{FB}) = 0$ for $\forall i$. The second-best level of efforts of the parent's children induced from solving (12) are smaller than $\mathbf{a}^{FB} = (a_1^{FB}, \cdots, a_n^{FB})$. To see this, there must exist a_i obtained from solving (12) is smaller than a_i^{FB}, due to (6), (9), (10), and $n > 1$, since the former meets $b_i f' - c' = 0$ with $b_i \in [0,1)$ while the latter meets $f' - c' = 0$. More importantly, any optimal inheritance rule $\mathbf{b}^* = (b_1^*, \cdots, b_n^*)$ that solves (12) cannot induce $\mathbf{a}^{FB} = (a_1^{FB}, \cdots, a_n^{FB})$ because there must exist $b_i \in [0,1)$ such that $b_i f' - c' = 0$, due to (6) and $n > 1$.

III. Characterization of Equilibria

Although the first-best outcome is not obtainable, the parent still can induce the second-best level of efforts from his children by announcing a reward scheme that maximizes U_p. Because, under the no-strike condition (10), all sets of $(\mathbf{a},\mathbf{b})$ that satisfy (9) and (11) always meet the IC constraints, we can further simplify the parent's problem (12) into

(14) $$\max_{\mathbf{b}\in\Delta^n} \sum_{i=1}^{n} f(\gamma^{-1}(b_i)) - \sum_{i=1}^{n} c(\gamma^{-1}(b_i))$$

whose first derivative yields

(15) $$(f' - c')\frac{1}{\gamma'} \geq 0 \text{ for } \forall i$$

due to (6), (8), (10), and $f' > 0$. On the other hand, however, the sign of the second derivative of (14), $(f'' - c'')\frac{1}{\gamma'} - (f' - c')\frac{\gamma''}{(\gamma')^2}$, is not pinned down, at the present level of generality. Nevertheless, the number of all the possible cases is only three depending on the sign, so we can seek to comprehensively characterize all the possible equilibrium inheritance rules $\mathbf{b}*$ by examining the three cases. To do this in an informative way, based on (10) and (15), we rewrite the sign of the second derivative of (14) as

(16) $$\text{sign}\{(f'' - c'')\frac{1}{\gamma'} - (f' - c')\frac{\gamma''}{(\gamma')^2}\} = -\text{sign}\{(-\frac{f'' - c''}{f' - c'}) - (-\frac{\gamma''}{\gamma'})\}$$

Notice that $f' - c'$ is the marginal benefit (marginal surplus in family production) from an increment in b_i by inducing an infinitesimal increase in efforts for family production, while γ' is the marginal cost of the increment in b_i paid to induce the increase in efforts for family production. As these two terms ($f' - c'$ and γ') are measured in different units, they cannot be directly compared; however, their unit-free rates of dwindling or growing can be properly compared. Namely, as

$\frac{f''-c''}{f'-c'}<0$, $-\frac{f''-c''}{f'-c'}$ is the rate at which the marginal benefit *grows* as b_i marginally increases and induces infinitesimal changes in efforts for family production; and, in this line, $-\frac{\gamma''}{\gamma'}$ is the rate at which the marginal cost *grows* as the bequest share b_i marginally increases. Thus, $\frac{f''-c''}{f'-c'}$ is the rate at which the marginal benefit *dwindles*, while $\frac{\gamma''}{\gamma'}$ is the rate at which the marginal cost *dwindles*. Most of all, due to (6) (equivalently, by the nature of *n*-dimensional simplex Δ^n), a marginal increase in b_i is *always* ensued by a marginal decrease in bequest share of another child.

Basically, the parent begins his search for an optimal inheritance rule $\mathbf{b}^*$ with an initial point $\mathbf{u} \in \Delta^n$ where $\mathbf{u}$ is an *n*-dimensional vector whose elements are all zero except for the *j*th element that takes the value of one, for a given *j*. The parent then compares $U_p(\mathbf{u})$ the payoff at the initial point $\mathbf{u}$ (giving all to child *j*) with a payoff at a different alternative point which entails an increment in the *k*th element b_k, for some $k \neq j$, from zero at the cost of the equal amount of decrement in b_j from one. Whether this deviation from $\mathbf{u}$ brings a net gain or loss depends on whether the growth rate of the marginal benefit is greater than the growth rate of the marginal cost or not (i.e., $\text{sign}\{(-\frac{f''-c''}{f'-c'})-(-\frac{\gamma''}{\gamma'})\}$). While the increment in the bequest share to a zero-share child *k* is of the *same* size as the entailed decrement in the bequest share to the sole inheritor child *j*, this reallocation draws effort-responses of *different* degrees, in light of (10) and (11), due to the difference in their current bequest share levels.[8] That is, the induced increase in the effort of child *k* may be of larger or smaller size than the entailed decrease in the effort of child *j*. As a consequence, we can compare different marginal costs of bequest share paid to the different changes in the efforts, based on the best response function of (11), to calculate the growth rate of the marginal cost. At the same time, we can obtain the growth rate of the marginal benefit from the induced efforts by comparing different marginal benefits resulting from the changes in the efforts.

First, if the marginal benefit from inducing efforts from children grows more slowly than the marginal cost ($-\frac{f''-c''}{f'-c'}<-\frac{\gamma''}{\gamma'}$), a deviation from **u** will only bring a net loss and reduce the parent's payoff; therefore, the parent stays at **u** as an equilibrium. Second, in contrast, if the marginal benefit grows more quickly than the marginal cost ($-\frac{f''-c''}{f'-c'}>-\frac{\gamma''}{\gamma'}$), then the parent is better off with deviating from **u** and will keep moving to different points in Δ^n until he reaches a point where he has no further room to be better off. The parent will eventually arrive at the point of equal division $\mathbf{e}\in\Delta^n$, where **e** is an *n*-dimensional vector whose elements are all $\frac{1}{n}$, because a payoff from any other **b** that unequally distributes shares can be increased by reallocating bequest share from a larger-share child to a smaller-share child. Thirdly, if the marginal benefit and the marginal cost grow at the exactly same rate ($-\frac{f''-c''}{f'-c'}=-\frac{\gamma''}{\gamma'}$), which is clearly rarer than the other two cases, the parent is indifferent between deviating from **u**

[8] As the second derivative of the best response function (11) is not restricted to remain zero throughout all the possible values of bequest share, (11) is not a simple linear function. Thus, effort response to the same increment (decrement) in bequest share differs by the currently held bequest shares.

or choosing $\mathbf{u}$. Thus, the parent's optimal $\mathbf{b}^*$ will be indeterminate in this sharp-tie case.

Having described the rationale underlying the parent's search for an optimal inheritance rule, let us elaborate on the characteristics of equilibrium distribution of bequests $\mathbf{b}^*$ in each of the all possible three cases in the following propositions.

Proposition 1. When the marginal benefit from efforts of children for family production grows more slowly than the marginal cost of inducing the efforts from the children ($-\frac{f''-c''}{f'-c'} < -\frac{\gamma''}{\gamma'}$), unigeniture is the optimal inheritance rule. That is, when $-\frac{f''-c''}{f'-c'} < -\frac{\gamma''}{\gamma'}$, all bequests are given to only one child in equilibrium. Moreover, this equilibrium *distribution* of bequests is unique.

Proof. At first, unigeniture can be denoted by an n-dimensional vector $\mathbf{u} \in \Delta^n$ whose ith element is one, for an arbitrarily given $i \in \{1, \cdots, n\}$, having all the other elements as zero to meet (6). To prove that unigeniture $\mathbf{u}$ is optimal when $-\frac{f''-c''}{f'-c'} < -\frac{\gamma''}{\gamma'}$, it is enough to show that our maximand in (14), denoted by $U_p(\mathbf{b})$, is Schur-convex when $-\frac{f''-c''}{f'-c'} < -\frac{\gamma''}{\gamma'}$. In particular, according to Marshall et al. (2011),[9] we only need to show that when $-\frac{f''-c''}{f'-c'} < -\frac{\gamma''}{\gamma'}$, $(b_j - b_k)(U_{pj} - U_{pk}) > 0$ for any $b_j \neq b_k$ (where $U_{pi} = \frac{\partial U_p}{\partial b_i}$) because $U_p(\mathbf{b})$ is continuously differentiable due to (9), (10), and continuous differentiability of f and c. To this end, pick any $b_j \neq b_k$ in an arbitrary $\mathbf{b}$ from the domain defined by

[9] In particular, refer to the result in p.84 in Marshall et al. (2011). Moreover, Schur-convexity/Schur-concavity (p.80) and majorization (p.8) are described in Marshall et al. (2011). In brief, for any two vectors **X** and **Y** in the $\triangle^n$, we call **Y** majorizes **X** when the elements of **X** is more equally spread than those of **Y**. A function φ is Schur-convex (Schur-concave) if whenever **Y** majorizes **X**, φ(**X**) <φ(**Y**) (φ(**X**)>φ(**Y**) for Schur-concavity).

(6) (n-dimensional simplex Δ^n) and suppose that $b_j > b_k$ without loss of generality. Notice that U_{pi} increases as the value of b_i increases from b_k to b_j because the derivative of U_{pi}, $(f'' - c'')\frac{1}{\gamma'} - (f' - c')\frac{\gamma''}{(\gamma')^2}$, is positive, due to (16) and $-\frac{f'' - c''}{f' - c'} < -\frac{\gamma''}{\gamma'}$. This implies that $U_{pj} - U_{pk} > 0$ whenever $b_j - b_k > 0$. Therefore, $U_p(\mathbf{b})$ is Schur-convex. Given the Schur-convexity of U_p, because $\mathbf{u}$ majorizes any other vectors which are distributionally different from $\mathbf{u}$ in the n-dimensional simplex Δ^n, unigeniture $\mathbf{u}$ is the maximizer of U_p in (14). Thus, unigeniture $\mathbf{u}$ is the optimal inheritance rule (i.e., $\mathbf{b}^* = \mathbf{u}$) when $-\frac{f'' - c''}{f' - c'} < -\frac{\gamma''}{\gamma'}$.

Next, we want to prove that this equilibrium *distribution* is unique when $-\frac{f'' - c''}{f' - c'} < -\frac{\gamma''}{\gamma'}$. To show this, suppose that there exists another equilibrium $\mathbf{b}'$ ($\mathbf{b}' \neq \mathbf{u}$ in the n-dimensional simplex Δ^n) which maximizes U_p and is distributionally different from $\mathbf{u}$. Then, $\mathbf{b}'$ has at least two strictly positive elements, all of which are smaller than one, due to (6). First, this implies that $U_p(\mathbf{b}') \neq U_p(\mathbf{u})$ because of the strict monotonicity of f and c. Second, when we rank individual elements from the greatest to the smallest such that $b'_{[i]}$ (or $u_{[i]}$) is the ith largest element of $\mathbf{b}'$ (or $\mathbf{u}$), $\sum_{i=1}^{m} u_{[i]} = 1 > \sum_{i=1}^{m} b'_{[i]}$ for $\forall m < n$. In addition to this, $\sum_{i=1}^{n} u_{[i]} = \sum_{i=1}^{n} b'_{[i]} = 1$, because of (6), which means that $\mathbf{u}$ strictly majorizes $\mathbf{b}'$. By the property of Schur-convexity, this implies that $U_p(\mathbf{b}') < U_p(\mathbf{u})$. A contradiction to the assumption that $\mathbf{b}'$ is an equilibrium that maximizes U_p. So, given *i*, $\mathbf{u}$ is the unique equilibrium under $-\frac{f'' - c''}{f' - c'} < -\frac{\gamma''}{\gamma'}$. However, *i* is arbitrarily given in the first place, and the above proof also holds when we pick another child (instead of child *i*) as the sole inheritor. Therefore, the equilibrium *distribution* of giving all the bequests to one child is unique when $-\frac{f'' - c''}{f' - c'} < -\frac{\gamma''}{\gamma'}$. ■

Proposition 2. When the marginal benefit from efforts of children for family production grows more quickly than the marginal cost of inducing the efforts from the children ($-\frac{f''-c''}{f'-c'} > -\frac{\gamma''}{\gamma'}$), equigeniture is the optimal inheritance rule. That is, when $-\frac{f''-c''}{f'-c'} > -\frac{\gamma''}{\gamma'}$, bequests are divided equally among the children in equilibrium. Moreover, this equilibrium is unique.

Proof. At the outset, equigeniture is denoted by an n-dimensional vector $\mathbf{e} \in \Delta^n$ all of whose element is $\frac{1}{n}$. By the same token of the proof for **Proposition 1**, it is enough to show that $U_p(\mathbf{b})$ of (14) is Schur-concave by proving that $(b_j - b_k)(U_{pj} - U_{pk}) < 0$ for any $b_j \neq b_k$ when $-\frac{f''-c''}{f'-c'} > -\frac{\gamma''}{\gamma'}$. To this end, pick any $b_j \neq b_k$ in an arbitrary $\mathbf{b}$ from the domain defined by (6) (n-dimensional simplex Δ^n) and suppose that $b_j > b_k$ without loss of generality. Notice that U_{pi} decreases as the value of b_i increases from b_k to b_j because the derivative of U_{pi}, $(f''-c'')\frac{1}{\gamma'} - (f'-c')\frac{\gamma''}{(\gamma')^2}$, is negative, due to (16) and $-\frac{f''-c''}{f'-c'} > -\frac{\gamma''}{\gamma'}$. This implies that $U_{pj} - U_{pk} < 0$ whenever $b_j - b_k > 0$. Therefore, $U_p(\mathbf{b})$ is Schur-concave. Given the Schur-concavity of U_p, because $\mathbf{e}$ is majorized by any other vectors in the domain Δ^n, equigeniture $\mathbf{e}$ is the maximizer of U_p when $-\frac{f''-c''}{f'-c'} > -\frac{\gamma''}{\gamma'}$. Thus, equigeniture $\mathbf{e}$ is the optimal inheritance rule (i.e., $\mathbf{b}^* = \mathbf{e}$) when $-\frac{f''-c''}{f'-c'} > -\frac{\gamma''}{\gamma'}$.

Next, we want to prove that $\mathbf{e}$ is the unique equilibrium under $-\frac{f''-c''}{f'-c'} > -\frac{\gamma''}{\gamma'}$. To show this, suppose that there exists another equilibrium $\mathbf{b}' \neq \mathbf{e}$ which maximizes U_p. First of all, notice that $\mathbf{e}$ is majorized by $\mathbf{b}'$ because all the elements of $\mathbf{e}$ are constantly $\frac{1}{n}$. That is, the elements of $\mathbf{e}$ are more spread out than those of all the other vectors in the domain Δ^n including $\mathbf{b}'$. Secondly, $U_p(\mathbf{b}') \neq U_p(\mathbf{e})$ due to the strict monotonicity of f and c. Thirdly, $\sum_{i=1}^{n} e_{[i]} = \sum_{i=1}^{n} b'_{[i]} = 1$ because of (6). By the property of Schur-concavity, therefore, it follows that $U_p(\mathbf{b}') < U_p(\mathbf{e})$: A contradiction to the assumption that $\mathbf{b}'$ is an equilibrium that maximizes U_p. So, $\mathbf{e}$ is the unique equilibrium inheritance rule when $-\frac{f''-c''}{f'-c'} > -\frac{\gamma''}{\gamma'}$. ■

Thirdly, when the marginal benefit from efforts of children for family production grows at the precisely same rate as the marginal cost of inducing the efforts from the children ($-\frac{f''-c''}{f'-c'} = -\frac{\gamma''}{\gamma'}$), equilibrium inheritance rule is not clearly determined, in contrast to the preceding two cases, at the present level of generality. When $-\frac{f''-c''}{f'-c'} = -\frac{\gamma''}{\gamma'}$, $(b_j - b_k)(U_{pj} - U_{pk}) = 0$ for any $b_j > b_k$ in an n-dimensional vector $\mathbf{b}$ in the domain (simplex Δ^n) defined by (6), which implies that $(U_{pj} - U_{pk}) = 0$. Thus, distributing bequest share from a larger-share child to a smaller-share child does not change the parent's payoff (utility) U_p, which prevents the parent from selecting an equilibrium inheritance rule $\mathbf{b}^*$.

Notably, even after specifying the functions of f and c concretely, an equilibrium distribution of bequests in this 'knife-edge' case of $-\frac{f''-c''}{f'-c'} = -\frac{\gamma''}{\gamma'}$, if any, is not *stable* but vulnerable to small perturbations, as opposed to the other preceding two cases, since other distributions of bequests, adjacent to the equilibrium, may easily rise as an alternative equilibrium by giving equal payoffs to the parent that the original equilibrium gives. Furthermore, an equilibrium in the sharp-tie case of $-\frac{f''-c''}{f'-c'} = -\frac{\gamma''}{\gamma'}$, even if it exists, is also vulnerable to a small change in the parameters of the functions f and c, which easily breaks the equality $-\frac{f''-c''}{f'-c'} = -\frac{\gamma''}{\gamma'}$ and moves equilibrium inheritance rule to unigeniture or equigeniture, according to **Proposition 1** and **2**. The number of parameters' values that satisfy the inequality $-\frac{f''-c''}{f'-c'} > -\frac{\gamma''}{\gamma'}$ or $-\frac{f''-c''}{f'-c'} < -\frac{\gamma''}{\gamma'}$ can be infinitely large and is always much greater than the number of parameters' values that precisely meet the equality $-\frac{f''-c''}{f'-c'} = -\frac{\gamma''}{\gamma'}$, which is finite.

Taking all together, we find that only the two extreme distributions of bequests — unigeniture and equigeniture — constitute all the possible stable equilibrium inheritance rules. One could be puzzled that unigeniture, where only one child exerts effort, maximizes U_p arguing that U_p is concave with respect to individual efforts of the parent's children. However, recall that what the parent can choose is distribution of bequests $\mathbf{b}$, *not* individual efforts of his children. Having a peculiar domain, simplex Δ^n, U_p can be Schur-concave or Schur-convex (instead of only concave) with respect to his choice vector $\mathbf{b}$, depending on the values of parameters of f and c.

Under unigeniture, the sole inheritor provides first-best level of effort, as he is the claimant of full of what his own effort contributes for family production. As he is the only effort-provider for family production, the total output, which is observable in (iii), accurately reveals the level of his effort spent in (ii). To make more children work for family production, the parent may reallocate a bequest share to other children. While this deviation from unigeniture newly induces more efforts from another child, it reduces efforts of the sole inheritor more than the decrement in his bequest share from one.[10] As efforts of children for family production are not separately verifiable, each of individual inheritors *cannot claim full of what his own effort yields* for family production. Therefore, the amount of

[10] Bolton and Dewatripont (2005) explains under-provision of efforts in team work in the logic of under-provision of public good; since effort of an individual child for family production has positive externality to other children who are benefited from his effort via theirs share of total output.

efforts from the parent's children resulting from the deviation may or may not be effective for increasing his payoff, although the number of effort-providers is increased.

Most importantly, all the possible stable equilibrium distributions of bequests, which arise in our model, are unigeniture and equigeniture only, which is consistent with the observation that inheritance rules in hundreds of diverse societies across times and regions around of the world are polarized into the two extreme distributions of bequests, unigeniture and equigeniture only, in spite of the vast difference in economic and cultural environments that parents face.

Notice that polarization of inheritance rules of diverse societies was not explained by any of the previous studies. Bernheim and Severinov (2003) examined the conditions that engender equigeniture and unigeniture; however, they did not analyze all the possible (stable) equilibria rising from their model. Moreover, equigeniture observed in the large portion of different societies across times and regions is not well explained either. In both models of altruism (Becker 1974) and exchange motive (Bernheim et al. 1985), equigeniture is an *unstable* knife-edge case, instead of a stable optimal inheritance rule.

IV. Evolution of Stable Equilibrium

Having identified stable equilibrium inheritance rules with unigeniture and equigeniture, we analyze the evolution from the former to the latter. In the light of **Propositions 1** and **2**, what raises the growth rate of the marginal benefit from efforts can change the parent's choice from unigeniture to equigeniture, as it causes the marginal benefit of efforts of children for family production to grow more quickly than the marginal cost of inducing the efforts. Particularly, when a given amount of efforts produce more output in family production, a larger increase in the marginal benefit is obtained from the same efforts, which implies an increase in the growth rate of the marginal benefit. In other words, a rise in the productivity of efforts for family production can result in the evolution from unigeniture to equigeniture. To formally prove this, consider

(15) $$f(a_i) = \theta a_i^{\alpha} \text{ for } \forall i$$

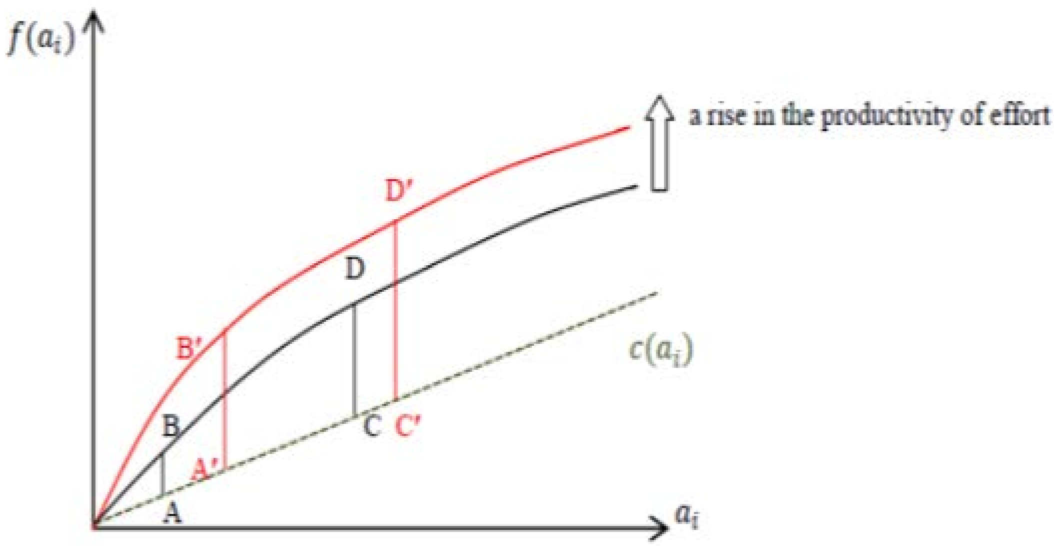
(16) $$c(a_i) = \phi a_i^{\beta} \text{ for } \forall i .$$

Note that $\theta > 0$ reflects productivity of efforts for family production; the higher value θ takes, the larger output of family production a given amount of efforts produces. Thus, by investigating whether an increase in θ raises $(-\frac{f''-c''}{f'-c'}) - (-\frac{\gamma''}{\gamma'})$ or not, we can find whether a rise in the productivity of efforts drives the evolution from unigeniture to equigeniture.

Proposition 3. When productivity of efforts for family production rises, equigeniture is chosen over unigeniture.

Proof. At the outset, note that $\phi > 0$ and $f' - c' = \theta\alpha a_i^{\alpha-1} - \phi\beta a_i^{\beta-1} \geq 0$, based on (10) and (15), and that due to concavity of f and convexity of c, $f'' - c''$ $= \theta\alpha(\alpha-1)a_i^{\alpha-2} - \phi\beta(\beta-1)a_i^{\beta-2} < 0$ and $0 < \alpha < 1$. Moreover, due to (9), $\gamma =$ $\frac{\phi\beta}{\theta\alpha} a_i^{\beta-\alpha}$, $\gamma' = \frac{\phi\beta(\beta-\alpha)}{\theta\alpha} a_i^{\beta-\alpha-1}$, and $\gamma'' = \frac{\phi\beta(\beta-\alpha)(\beta-\alpha-1)}{\theta\alpha} a_i^{\beta-\alpha-2}$. Hence, $(-\frac{f''-c''}{f'-c'}) - (-\frac{\gamma''}{\gamma'}) = -\frac{\theta\alpha(\alpha-1)a_i^{\alpha-2} - \phi\beta(\beta-1)a_i^{\beta-2}}{\theta\alpha a_i^{\alpha-1} - \phi\beta a_i^{\beta-1}} + (\beta-\alpha-1)a_i^{-1}$. Whether an increase in θ increases $(-\frac{f''-c''}{f'-c'}) - (-\frac{\gamma''}{\gamma'})$ is indicated by the sign of $\frac{\partial}{\partial\theta}[(-\frac{f''-c''}{f'-c'}) - (-\frac{\gamma''}{\gamma'})]$. Thus, we get $\frac{\partial}{\partial\theta}[(-\frac{f''-c''}{f'-c'}) - (-\frac{\gamma''}{\gamma'})] = \frac{-\alpha(\alpha-1)a_i^{\alpha-2}}{\theta\alpha a_i^{\alpha-1} - \phi\beta a_i^{\beta-1}} +$ $\frac{-\alpha a_i^{\alpha-1}[\theta\alpha(\alpha-1)a_i^{\alpha-2} - \phi\beta(\beta-1)a_i^{\beta-2}]}{[\theta\alpha a_i^{\alpha-1} - \phi\beta a_i^{\beta-1}]^2} > 0$ because concavity of f and convexity of c imply that the first term and the second term alike are positive. This means that a rise in θ raises the value of $(-\frac{f''-c''}{f'-c'}) - (-\frac{\gamma''}{\gamma'})$ *from* a negative one, which yields unigeniture as the optimal inheritance rule, according to **Proposition 1**, *to* a positive one, which yields equigeniture as the optimal inheritance rule, according to **Proposition 2**. On the other hand, since $\frac{f}{a_i} = \theta a_i^{\alpha-1}$ increases whenever θ increases, θ reflects productivity of efforts for family production. Therefore, from **Proposition 1** and **2**, a rise in the productivity of efforts from an increase in θ causes the evolution of the parent's choice from unigeniture to equigeniture. ■

Figure 1. Unigeniture vs. Equigeniture and Productivity of Efforts ($n = 2$)



Note: The slopes of the lines tangent to $f(a_i)$ at D and to $f(a_i)$ at D' are equal to the slope of $c(a_i)$ whereas those at B and B' are twice the slope of $c(a_i)$. The payoff to the parent U_p under unigeniture is $\overline{DC}$ and is $\overline{D'C'}$ after a rise in the productivity of efforts whereas U_p under equigeniture is $2\overline{AB}$ and is $2\overline{A'B'}$ after the rise. The rise in the productivity of efforts (from f to f) makes equigeniture more profitable than unigeniture, as $2\overline{AB} < \overline{DC}$ while $2\overline{A'B'} > \overline{D'C'}$.

For an illustration of **Proposition 3**, a corollary of **Propositions 1** and **2**, see **Figure 1**. The payoff to the parent under unigeniture, $U_p(\mathbf{u})$, is $\overline{DC}$ before a rise in the productivity of efforts and $\overline{D'C'}$ after the rise, where the slopes of the lines tangent to $f(a_i)$ at D and D' are equal to the slope of $c(a_i)$, as the sole inheritor exerts his first-best efforts. On the other hand, under equigeniture with $n = 2$, both inheritors exert second-best efforts; and, the benefit yielded by each inheritor is $\overline{AB}$ and $\overline{A'B'}$ where the slopes of the lines tangent to $f(a_i)$ at B and B' are twice as steep as the slope of $c(a_i)$ as $b_i = \frac{c'}{f'} = \frac{1}{2}$ based on (9). Thus, the payoff to the parent under equigeniture $U_p(\mathbf{e})$ is $2\overline{AB}$ before the rise in the productivity of efforts and $2\overline{A'B'}$ after the rise. Most importantly, the rise in the productivity of efforts, which is depicted as shift to the steeper $f(a_i)$ in **Figure 1**, causes equigeniture to turn more profitable than unigeniture, because $2\,\overline{AB} < \overline{DC}$ whereas $2\overline{A'B'} > \overline{D'C'}$.

Applying **Proposition 3** to pre-industrial agrarian societies, where the main function of family was subsistence, higher agricultural labor productivity makes equigeniture be more likely to be adopted than unigeniture. This may explain why peasant parents in feudal England followed primogeniture (Goody et al. 1978) until 1926, while their counterparts in feudal China (Wakefield 1998) or India (Sharma 2003) already adopted equigeniture, although social mobility of feudal China or India is not greater than feudal England. With intensive agriculture, agricultural labor productivities of feudal China and India were higher than feudal England that left one third of arable land as fallow (Grigg 1974).

Another important application of **Proposition 3** is that equigeniture in industrialized societies stems from change in the main function of family to emotional support, over the industrialization, which transforms effort for family from manual one to psychological one. As a result of industrialization, the process of family production evolved from manual one to psychological one, yielding intangible output such as deep, special bonding and prestige and esteem (Becker 1981). Notably, the non-physical process of producing emotional betterment of family is not constrained by physical production technology and thus enables marginal returns of efforts to diminish significantly less than the physical process of producing food for subsistence of family. Consequently, in industrialized societies, performing the main function of family well can be achieved by family members with spending most of their time at their work places outside the family, whereas producing food for family needs full-time efforts of family members at their family land. Therefore, a rise in the productivity of efforts was ensued by the change in the principal function of family over the industrialization, leading the evolution to equigeniture.

More importantly, notice that once rises in the productivity of efforts (such as from increases in agricultural labor productivity and from change in the main function of family) occurred, regress of undoing the rises did not happen in history, which explains the observed irreversity of the evolutionary trend from unigeniture to equigeniture in light of **Proposition 3**.

V. Concluding Remarks

In sum, this paper presents a theoretical model of bequest that coherently explains observed bequest patterns across vastly diverse societies of different times and regions: (i) polarization of inheritance rules into unigeniture and equigeniture, and (ii) evolutionary trend from unigeniture to equigeniture. As a leader of a family team, a parent wants his children to exert costly but unverifiable efforts for fulfilling the main function of family, while he cares about his children. The parent ends up with one of the only two stable equilibrium inheritance rules: unigeniture or equigeniture, which rationalizes (i). Moreover, we find that a change from unigeniture to equigeniture is led by a rise in the productivity of efforts for family, which is consistent with (ii). This finding from our model can also explain adoption of equigeniture in feudal China or medieval India which was less egalitarian than the 19th century England that followed primogeniture (a type of unigeniture).

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